CHAPTER 3:

Analyze Optimal Solutions: Sensitivity Analysis

Section 3.0: Introduction

In addition to solving problems, operations researchers are often interested in learning how sensitive their solutions are to changes in the parameters of the problem. Consider the Computer Flips problem in the previous chapter. How sensitive is the solution to changes in the amount of profit that is made on each type of computer? What would be the effect of increasing the amount of available installation time or testing time? Questions such as these are part of what is called **sensitivity analysis**.

Section 3.1: Computer Flips, a Junior Achievement Company

Recall from Chapter 2 that Computer Flips is a Junior Achievement Company that begins producing two computer models: Simplex and Omniplex. The pertinent data from the Computer Flips problem appear in Table 3.1.1.

	Simplex	Omniplex
Profit per Computer	\$200	\$300
Installation Time per Computer	60 min.	120 min.

Table 3.1.1: Computer Flips information for two computer types

In addition, Computer Flips has 2,400 min of installation time available per week (five students, each working eight hours per week). They are also under two market restrictions. They estimate that they cannot sell more than 20 Simplex computers or 16 Omniplex computers per week. Gates Williams, the production manager for Computer Flips, wants to find the production rate per week for each type of computer that will maximize total profit.

3.1.1 Problem Formulation

Gates Williams writes the complete linear programming formulation for this problem. He begins with the definition of the decision variables, then he writes the objective function, and finally he lists the constraints.

<u>Decision Variables</u>

Let: x_1 = the weekly production rate of Simplex computers
x_2 = the weekly production rate of Omniplex computers

<u>Objective Function</u>

Maximize: $z = 200x_1 + 300x_2$, where z = Computer Flips' weekly profit

<u>Constraints</u>

Subject to:

Installation Time (min): $60x_1 + 120x_2 \leq 2400$
Simplex Market (#): $x_1 \leq 20$
Omniplex Market (#): $x_2 \leq 16$
Non-Negativity: $x_1 \geq 0$ and $x_2 \geq 0$

Q1. Using Excel Solver, find the optimal solution to the Computer Flips problem.
a. How many Simplex and Omniplex computers should be produced per week?
b. How much profit will Computer Flips earn per week?

3.1.2 Solver Answer Report

To begin to analyze this solution, Gates Williams has Solver generate the Answer and Sensitivity Reports, as shown in Figure 3.1.1. He starts by looking at the Answer Report Figure

3.1.2. He notices that the optimal solution and the value of the objective function are listed on this report.

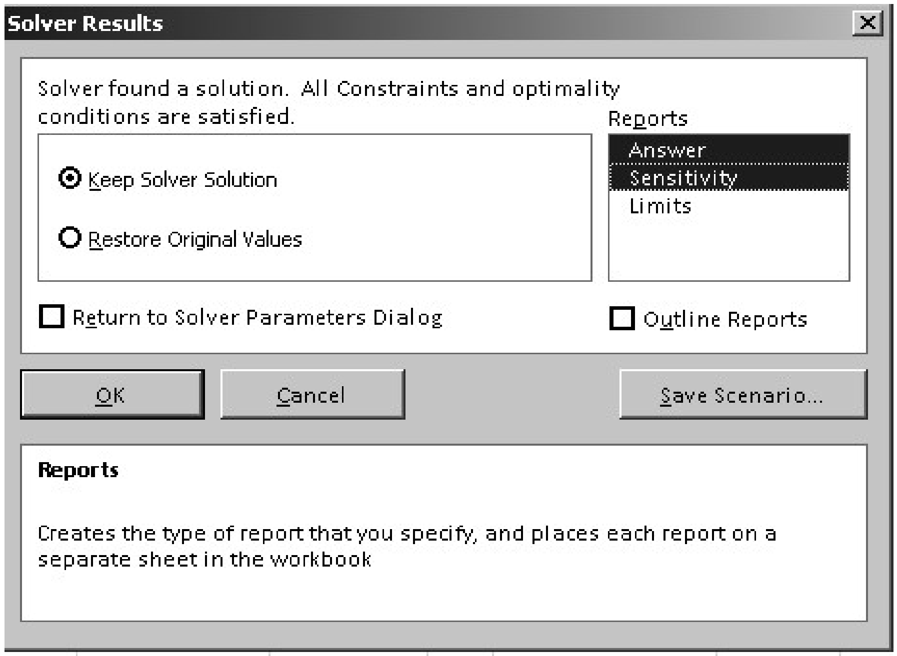


Figure 3.1.1: Choosing Answer and Sensitivity Reports

	A	B	C	D	E	F	G
1	**Microsoft Excel 14.0 Answer Report**						
2	**Worksheet: [Ch_2 computer flips.xlsx]2 variables**						
3	**Report Created: 12/5/2013 11:13:50 AM**						
4	**Result: Solver found a solution. All Constraints and optimality conditions are satisfied.**						
5							
6	Objective Cell (Max)						
7		**Cell**	**Name**	**Original Value**	**Final Value**		
8		$D$7	objective function Total Profit	0	7000		
9							
10							
11	Variable Cells						
12		**Cell**	**Name**	**Original Value**	**Final Value**	**Integer**	
13		$B$5	decision variable values x1	0	20	Contin	
14		$C$5	decision variable values x2	0	10	Contin	
15							
16							
17	Constraints						
18		**Cell**	**Name**	**Cell Value**	**Formula**	**Status**	**Slack**
19		$D$9	installation time constraint Total Profit	2400	$D$9<=$F$9	Binding	0
20		$D$10	market 1 Total Profit	20	$D$10<=$F$10	Binding	0
21		$D$11	market 2 Total Profit	10	$D$11<=$F$11	Not Binding	5

Figure 3.1.2: Computer Flips Answer Report

Q2. How could you determine the optimal solution and objective function value by looking *only* at the Answer Report?

Next, Gates Williams looks at the Constraints portion of the Answer Report. He notices constraints are either listed as Binding or Not Binding. He also notices that there is a column for Slack.

Q3. In the context of this problem, what does the information given in the Cell Value column mean?

Q4. Connect your response to the previous question to the amount of constraints available.
  a. Based on this information, what do you think *Binding* means?
  b. Based on this information, what do you think *Slack* means?
  c. If you were only given the slack value for a constraint, how could you determine whether that constraint is binding?

Gates Williams is considering asking each student to work an additional hour each week. This would increase the available installation time by: (5 students)(1 hour)(60 minutes) = 300. Therefore, the new total installation time available would be 2,700 minutes.

Q5. In your Excel worksheet, change the right hand side of the installation time constraint to 2700 minutes. Solve the problem again and open the Answer Report.
  a. What changes do you observe?
  b. Do you think it is worth it for Gates Williams to ask the students to work this extra time each week?

Gates Williams decides that he will not ask students to work this extra hour each week. Instead, he wonders if an increase in the Omniplex marketability will increase the weekly profit.

Q6. In your Excel worksheet, change the right hand side of the installation time constraint back to 2400 minutes. Then, increase the Omniplex marketability constraint to 17. Solve the problem again and open the Answer Report.
  a. What changes do you observe from the original problem?
  b. What happens if the Omniplex marketability constraint is 20 computers? 50 computers?
  c. Do you think Gates Williams should try to increase the marketability of the Omniplex computer? Why or why not?

3.1.3 Solver Sensitivity Report: Variable Cells

Now, Gates Williams opens the Sensitivity Report, Figure 3.1.3. He notices that the report is split into two sections: Variable Cells and Constraints. (Note: in older versions of Microsoft Office, Variable Cells are referred to as Adjustable Cells.)

In general, **Variable Cells** tell Gates Williams how the objective function coefficients may change. More specifically, Variable Cells show the increase or decrease of an objective function coefficient without changing the optimal solution. These changes only apply to one objective function coefficient at a time (all other coefficients must remain constant).

Next, the **Constraints** section tells Gates Williams how the objective function value changes. Specifically, Constraint cells give the objective function value based upon an increase or decrease of the right hand side (RHS) of a constraint. These changes only apply to one RHS constraint at a time (all other RHS constraints must remain the same).

Gates Williams feels that he only has control over how profitable each computer is. That is, he cannot change any of the constraints, but he could consider increasing the price of a computer. Therefore, he decides to only explore the Variable Cells in the Sensitivity Report.

Microsoft Excel 14.0 Sensitivity Report
Worksheet: [Ch_2 computer flips.xlsx]2 variables
Report Created: 12/5/2013 11:13:50 AM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	decision variable values x1	20	0	200	1E+30	50
$C$5	decision variable values x2	10	0	300	100	300

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$9	installation time constraint Total Profit	2400	2.5	2400	600	1200
$D$10	market 1 Total Profit	20	50	20	20	10
$D$11	market 2 Total Profit	10	0	15	1E+30	5

Figure 3.1.3: Computer Flips Sensitivity Report

The information in the Variable Cells section of the Sensitivity Report tells how sensitive the optimal solution is to changes in the objective function coefficients of the decision variables. Solver considers changes made to one coefficient at a time. In particular, **Allowable Increase** refers to how much the objective coefficient can be increased without changing the final values. Similarly, **Allowable Decrease** tells how much the objective coefficient can be decreased without changing the final values.

For now, Gates Williams only concerns himself with the Allowable Increase column of the Variable Cells section. He considers the coefficient of x_2, which is the amount of profit generated by the sale of one Omniplex computer. Currently, that profit is $300 per computer. He sees that the Allowable Increase for the coefficient of x_2 is $100.

Gates Williams is curious about the effect of increasing the profit per Omniplex computer. He considers increasing the profit by a value below the Allowable Increase, above the Allowable Increase, and exactly at the Allowable Increase. Thus, he explores the effect of increasing the profit by $50 (below the Allowable Increase), $200 (above the Allowable Increase), and $100 (exactly the Allowable Increase). The corresponding new objective function coefficients of x_2 are $350, $500, and $400, respectively. These changes affect only the objective function in the formulation of the problem. All of the constraints remain the same.

Q7. Write the new objective functions for each of these three changes.

First, Gates Williams considers increasing the profit of Omniplex computers so that the profitability is now \$350. However, he notices that this increase in profitability, \$50, is less than the Allowable Increase.

Q8. In your Excel worksheet, change the objective function coefficient for Omniplex computers to \$350. Solve the problem again.
   a. What changes to do you observe from the original problem?
   b. Do you think Gates Williams should try to increase the profitability of the Omniplex computer by \$50? Why or why not?

Next, Gates Williams looks at the effect of increasing the profitability of Omniplex by more than the Allowable Increase. He increases the profitability by \$200.

Q9. In your Excel worksheet, change the objective function coefficient for Omniplex computers to \$500. Solve the problem again.
   a. What changes to do you observe from the original problem?
   b. Do you think Gates Williams should try to increase the profitability of the Omniplex computer by \$200? Why or why not?

At this point, Gates Williams has looked at increasing the profitability of Omniplex by \$50, which is less than the Allowable Increase, and by \$200, which is more than the Allowable Increase. He wonders what would happen if the profitability of Omniplex increases by exactly \$100, which is the Allowable Increase.

Q10. In your Excel worksheet, change the objective function coefficient for Omniplex computers to \$400. Solve the problem again.
   a. What changes to do you observe from the original problem?
   b. Did your classmates obtain the same optimal solution as you?
   c. Do you think Gates Williams should try to increase the profitability of the Omniplex computer by \$100? Why or why not?

In order to gain a better understanding of the three examples explored above, Gates Williams considers the geometry of the situation.

Q11. Draw a graph of the feasible region for the original problem, including the original line of constant profit ($z = 200x_1 + 300x_2$) passing through the optimal corner point (refer to Section 2.1).

Q12. Draw another graph of the feasible region for the original problem.
   a. Draw the line of constant profit when the profitability of the Omniplex computer has increased by \$50 to \$350. Which corner point maximizes profit in this situation?
   b. Draw the line of constant profit when the profitability of the Omniplex computer has increased by \$200 to \$500. Which corner point maximizes profit in this situation?

c. Draw the line of constant profit when the profitability of the Omniplex computer has increased by \$100 to \$400. Which corner point maximizes profit in this situation? How is this situation different from the previous two?

Q13. Consider the case where the profit margin on Omniplex is increased to \$400.
a. What is the profit if 8 Simplex and 16 Omniplex computers are produced?
b. What is the profit if 20 Simplex and 10 Omniplex computers are produced?
c. Are there are other feasible points that produce this profit? If so, where are they? If not, why not?

Q14. Based on what you saw in this section, describe what you think would happen if you considered the Allowable Decrease instead.
a. How do you think the final values would change if Gates Williams decreased the profitability of Omniplex computers by \$200?
b. How do you think the final values would change if Gates Williams decreased the profitability of Omniplex computers by \$400?
c. How do you think the final values would change if Gates Williams decreased the profitability of Omniplex computers by \$300?
d. Put these changes into Excel to see if your predictions were correct.

Finally, Gates Williams notices that the Sensitivity Report shows an Allowable Increase of 1E+30 in the coefficient of x_1 in the objective function. The number 1E+30 is Solver's way of conveying the expression $1 \cdot 10^{30}$. This very large number is the best Solver can do to indicate an infinite Allowable Increase.

To understand why the Allowable Increase is infinite, Gates Williams first needs to think about what the coefficient of x_1 in the objective function represents. It is the profit margin on Simplex computers. Solver is showing that no matter how much the profitability of Simplex computers increases, it will not change the optimal solution. In other words, increasing the profit margin on Simplex computers is not going to change the optimal number to make. This makes sense because the optimal solution shows that to maximize profits, 20 Simplex (and 10 Omniplex) computers should be made. Twenty is the most that can be made and still satisfy the market limit on weekly sales of Simplex computers. Increasing the profit margin of Simplex computers will not change the fact that no more than 20 per week can be sold, and they're already making 20 each week.

In this section, we have explored the effects on the optimal solution of increasing or decreasing the profitability of one of the computer models. In reality, the situation is much more complicated. For example, if Computer Flips increased the price of an Omniplex computer by \$100-\$200 to make it more profitable, doing so might affect the Omniplex market constraint. The increased price might lower the market constraint. So, in practice, a company would try to explore all of the ramifications of making changes in the important parameters of the problem.

Section 3.2: SK8MAN, Inc.

Recall from Chapter 2 that SK8MAN, Inc. manufactures skateboards. G.F. Hurley, the production manager at SK8MAN, Inc., needed to determine the production rate for each type of skateboard in order to make the most profit. Table 3.2.1 shows the relevant data from Chapter 2.

	Sporty	Fancy	Pool Runner	Amount Available
Profit per skateboard	\$15	\$35	\$20	
Shaping time required	5	15	4	2,400 minutes
Truck availability	2	2	2	700 trucks
North American maple veneers required	0	7	0	840 veneers
Chinese maple veneers required	7	0	7	1,470 veneers

Table 3.2.1: SK8MAN, Inc. data

3.2.1 Problem Formulation

In Chapter 2, the following problem formulation was developed:

Decision Variables

Let: x_1 = the weekly production rate of Sporty boards
x_2 = the weekly production rate of Fancy boards
x_3 = the weekly production rate of Pool Runners boards

Objective Function

Maximize: $z = 15x_1 + 35x_2 + 20x_3$, where z = Computer Flips' weekly profit

Constraints

Subject to:

Shaping Time (min): $5x_1 + 15x_2 + 4x_3 \leq 2,400$

Trucks (#): $2x_1 + 2x_2 + 2x_3 \leq 700$

North American Maple (#): $7x_2 \leq 840$

Chinese Maple (#): $7x_1 + 7x_3 \leq 1,470$

Non-Negativity: $x_1 \;,\; x_2 \;,\; x_3 \geq 0$

3.2.2 Solver Answer Report

A spreadsheet formulation of the problem and an Answer Report showing the optimal solution appear in Figures 3.2.1 and 3.2.2.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	**0**	**104**	**210**			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	35	20			**$7,840.00**
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	**2400**	≤	2400
12	Truck Availability	2	2	2	**628**	≤	700
13	North American Maple Veneers	0	7	0	**728**	≤	840
14	Chinese Maple Veneers	7	0	7	**1470**	≤	1470

Figure 3.2.1: Formulation for the 3-decision variable SK8MAN problem

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$G$8	Objective Function [Profit ($)] Total Profit	$0.00	$7,840.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [# to make per week] Sporty (x1)	0	0	Contin
$C$6	Decision Values [# to make per week] Fancy (x2)	0	104	Contin
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	0	210	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$11	Shaping Time (minutes)	2400	$E$11<=$G$11	Binding	0
$E$12	Truck Availability	628	$E$12<=$G$12	Not Binding	72
$E$13	North American Maple Veneers	728	$E$13<=$G$13	Not Binding	112
$E$14	Chinese Maple Veneers	1470	$E$14<=$G$14	Binding	0

Figure 3.2.2: Answer Report for the 3-variable SK8MAN problem

As seen in Figure 3.2.1, the optimal solution is $x_1 = 0$, $x_2 = 104$, and $x_3 = 210$; that is, G.F. Hurley should produce no Sporty boards, 104 Fancy boards, and 210 Pool-Runner boards per week. From this product mix, SK8MAN will make a total profit of $7,840 each week. This information also appears in the Answer Report shown in Figure 3.2.2, along with some information about the constraints.

Examining the Answer Report, the first section refers to the Objective Cell (Max), and the variable name is "Objective Function [Profit ($)] Total Profit." The target cell in the spreadsheet, G8, stores the value of the objective function. Solver finds the maximum profit (because "Max" was selected during the Solver Parameters set-up) that meets all of the constraints. The maximum total profit Solver reports under the column Final Value is $7,840.00. The Original Value of $0.00 simply refers to the amount that was in the cell before Solver was run.

The second section is labeled Variable Cells and refers to the decision variables, x_1, x_2, and x_3. These are adjusted as Solver searches for the optimal solution. The Final Values of 0, 104, and 210, respectively, are the optimal solution. That is, the optimal solution is $x_1 = 0$, $x_2 = 104$, and $x_3 = 210$.

The third section of the Answer Report is labeled Constraints. The four constraints are all less than or equal to (≤) constraints. The left hand side value of each constraint represents the total amount used by the production plan. These totals are stored in cells E11, E12, E13, and E14 and are reported in the column labeled "Cell Value." The right hand side values for each of the three constraints are stored in column G in cells G11, G12, G13, and G14.

G.F. Hurley notices that of the four constraints, two of them are binding and two are not binding. But, he wonders what this means.

He notices that for the two binding constraints, there is a 0 in the column labeled Slack. He looks back to the Solver solution in Figure 3.2.1 and notices that for each of the two binding constraints, the left hand side of the constraint is equal to the right hand side.

For example, the workers at SK8MAN will use 5(0) + 15(104) + 4(210) = 2400 minutes for shaping (cell E11). They have 2400 minutes available for shaping (cell G11). In addition, they will use 7(0) + 7(210) = 1470 Chinese maple veneers (cell E14). They have 1470 Chinese maple veneers available (cell G14).

In other words, there is no slack because every bit of each of those resources is being used up by the optimal solution.

However, for the non-binding constraints in the Answer Report, the Slack values are not zero. They are listed as 72 and 112. Again returning to the Solver solution in Figure 3.1.1, G.F. Hurley notices that the left hand side of the truck availability constraint is 628, and the right hand side is 700. This time the two sides of the constraint are not equal because the optimal solution does not use up all available trucks. There is a Slack of 700 – 628 = 72. That means that SK8MAN could use 72 more trucks. They will not do that, though, because in order to use these extra trucks, they would have to make more skateboards, which is impossible due to the shaping time and the Chinese maple veneers constraints.

Similarly, G.F. Hurley notices that the left hand side of the North American maple veneers constraint is 728 and the right hand side is 840. Thus, they have an extra 840 – 728 = 112 North American maple veneers available.

3.2.3 Solver Sensitivity Report: Variable Cells

Figure 3.2.3 contains the Variable Cells section of the Sensitivity Report SK8MAN. Use it to answer the questions that follow.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$6	Decision Values [# to make per week] Sporty (x1)	0	-7.333333333	15	7.333333333	1E+30
$C$6	Decision Values [# to make per week] Fancy (x2)	104	0	35	40	35
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	210	0	20	1E+30	7.333333333

Figure 3.2.3: Sensitivity Report for the 3-variable SK8MAN problem

The Sensitivity Report provides information about each of the decision variables, which it calls Variable Cells. It also provides information about each of the constraints.

Variable Cells: Allowable Increase

The information provided in the Adjustable Cells section of the Sensitivity Report tells how sensitive the optimal solution is to changes in the objective function coefficients of the decision variables. Solver considers changes made to one coefficient at a time.

G.F. Hurley notices that for x_2, there is an Allowable Increase of 40. But he wonders what this refers to. What can be increased by 40? What does such an increase “allow”? Allowable Increase refers to increasing the coefficient of the decision variable in the objective function. In this case, the coefficient of x_2 is the amount of profit generated by the sale of one Fancy skateboard. Currently, that profit is $35 per skateboard.

G.F. Hurley is curious about the effect of increasing the profit per Fancy skateboard. He first explores the effect of increasing the profit by a value below the Allowable Increase. He chooses to increase the profit of Fancy boards by $25 to $60. Doing so changes the objective function to:

$$z = 15x_1 + 60x_2 + 20x_3.$$

The effect of this change is shown in Figures 3.2.4a and 3.2.4b.

G.F. Hurley notices that in Figure 3.2.4, the profitability of Fancy boards has been changed to $60. However, when looking at the Answer Report for that change, he sees that the optimal solution is the same: make Sporty boards at the rate of 0 per week, make Fancy boards at the rate of 104 per week, and make Pool-Runner boards at the rate of 210 per week.

Therefore, because G.F. Hurley increased the objective function coefficient by an amount *less than* the Allowable Increase, there was no change in the optimal solution. Although the optimal solution is the same, the increase in Omniplex profitability did change the weekly profit from $7,840 to $10,440. This $2,600 increase results from the $25 increase in profit on Fancy boards, and SK8MAN makes 104 per week: ($25)(104 Fancy boards) = $2600.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	**0**	**104**	**210**			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	60	20			**$10,440.00**
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	**2400**	≤	2400
12	Truck Availability	2	2	2	**628**	≤	700
13	North American Maple Veneers	0	7	0	**728**	≤	840
14	Chinese Maple Veneers	7	0	7	**1470**	≤	1470

Figure 3.2.4a: Formulation when the profitability of Fancy boards is $60

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$G$8	Objective Function [Profit ($)] Total Profit	$0.00	$10,440.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [# to make per week] Sporty (x1)	0	0	Contin
$C$6	Decision Values [# to make per week] Fancy (x2)	0	104	Contin
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	0	210	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$11	Shaping Time (minutes)	2400	$E$11<=$G$11	Binding	0
$E$12	Truck Availability	628	$E$12<=$G$12	Not Binding	72
$E$13	North American Maple Veneers	728	$E$13<=$G$13	Not Binding	112
$E$14	Chinese Maple Veneers	1470	$E$14<=$G$14	Binding	0

Figure 3.2.4b: Answer Report when the profitability of Fancy boards is $60

Next, G.F. Hurley increases the profitability of Fancy boards by an amount greater than the Allowable Increase. Figures 3.2.5a and 3.2.5b show the effect of increasing the objective function coefficient from $35 to $80 (an increase of $45), so that the objective function would now be:

$$z = 15x_1 + 80x_2 + 20x_3$$

In this case, G.F. Hurley sees that changing the profitability of Fancy boards to $80 yields a new optimal solution. This time, the optimal solution changes the production rates to 0 Sporty boards, 120 Fancy boards, and 150 Pool-Runner boards per week. Also, the amount of profit generated has now increased to $12,600 per week.

At this point, G.F. Hurley sees that increasing the profitability of Fancy boards by an amount *less than* the Allowable Increase has no impact on the optimal solution. However, increasing the profitability by an amount *greater than* the Allowable Increase changes the optimal solution. Next, he wonders what will happen if he increases the profitability by exactly the Allowable Increase.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	**0**	**120**	**150**			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	80	20			**$12,600.00**
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	**2400**	≤	2400
12	Truck Availability	2	2	2	**540**	≤	700
13	North American Maple Veneers	0	7	0	**840**	≤	840
14	Chinese Maple Veneers	7	0	7	**1050**	≤	1470

Figure 3.2.5a: Formulation when the profitability of Fancy boards is $80

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$G$8	Objective Function [Profit ($)] Total Profit	$0.00	$12,600.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [# to make per week] Sporty (x1)	0	0	Contin
$C$6	Decision Values [# to make per week] Fancy (x2)	0	120	Contin
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	0	150	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$11	Shaping Time (minutes)	2400	$E$11<=$G$11	Binding	0
$E$12	Truck Availability	540	$E$12<=$G$12	Not Binding	160
$E$13	North American Maple Veneers	840	$E$13<=$G$13	Binding	0
$E$14	Chinese Maple Veneers	1050	$E$14<=$G$14	Not Binding	420

Figure 3.2.5b: Answer Report when the profitability of Fancy boards is $80

Figures 3.2.6a and 3.2.6b show the effect of increasing the profitability of Fancy boards by exactly \$40, where the objective function is:

$$z = 15x_1 + 75x_2 + 20x_3$$

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	**0**	**120**	**150**			
7							**Total Profit**
8	Objective Function [Profit (\$)]	15	75	20			**\$12,000.00**
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	**2400**	≤	2400
12	Truck Availability	2	2	2	**540**	≤	700
13	North American Maple Veneers	0	7	0	**840**	≤	840
14	Chinese Maple Veneers	7	0	7	**1050**	≤	1470

Figure 3.2.6a: Formulation when the profitability of Fancy boards is \$75

Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$G\$8	Objective Function [Profit (\$)] Total Profit	\$0.00	\$12,000.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$6	Decision Values [# to make per week] Sporty (x1)	0	0	Contin
\$C\$6	Decision Values [# to make per week] Fancy (x2)	0	120	Contin
\$D\$6	Decision Values [# to make per week] Pool-Runner (x3)	0	150	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$E\$11	Shaping Time (minutes)	2400	\$E\$11<=\$G\$11	Binding	0
\$E\$12	Truck Availability	540	\$E\$12<=\$G\$12	Not Binding	160
\$E\$13	North American Maple Veneers	840	\$E\$13<=\$G\$13	Binding	0
\$E\$14	Chinese Maple Veneers	1050	\$E\$14<=\$G\$14	Not Binding	420

Figure 3.2.6b: Answer Report when the profitability of Fancy boards is \$75

G.F. Hurley notices that when the profitability of Fancy boards increases by exactly \$40 (to \$75), the optimal solution reported by Solver is to produce 0 Sporty boards, 120 Fancy boards, and 150 Pool-Runner boards. This solution is the same as when the profitability of Fancy boards was \$80. The total profit is now \$12,000:

$$15(0) + 75(120) + 20(150) = \$12,000.$$

However, he also notices that if the profitability of Fancy boards goes up to \$75, the amount of weekly profit generated by the original optimal solution (0 Sporty boards, 104 Fancy boards, and 210 Pool-Runner boards) is also \$12,000:

$$15(0) + 75(104) + 20(210) = \$12,000.$$

Both production plans lie on the same plane: $z = 15x_1 + 75x_2 + 20x_3$. In fact, any point that lies on this plane and is within the feasible region is an optimal solution. Therefore, there are infinitely many optimal solutions when the profitability of Fancy boards is \$75 (i.e., when the coefficient of x_2 is increased exactly by the amount of the Allowable Increase).

Note: This idea was explored graphically in the previous section, where the objective function was a line, rather than a plane. Visualizing the SK8MAN problem graphically is much more difficult because it has 3 decision variables.

Finally, G.F. Hurley notices that the Sensitivity Report in Figure 3.2.3 shows an Allowable Increase of 1E+30 in the coefficient of x_3 in the objective function. The number 1E+30 is Solver's way of expressing the number 1×10^{30}. This very large number is the best Solver can do to indicate an *infinite* Allowable Increase.

To understand why the Allowable Increase is infinite, G.F. Hurley first needs to think about what the coefficient of x_3 in the objective function represents. It is the profitability of Pool-Runner boards. Solver is showing that no matter how much the profitability of Pool-Runner boards increases, it will not change the optimal solution. In other words, increasing the profitability of Pool-Runner boards is not going to change the optimal number to make.

This makes sense because the optimal solution shows that to maximize profits, 210 Pool-Runner boards (as well as 0 Sporty boards and 104 Fancy boards) should be made. Making 210 Pool-Runner boards per week consumes 1470 Chinese maple veneers, which is exactly the number available per week. Therefore, no more than 210 Pool-Runner boards can be made per week, no matter how much their profitability increases. SK8MAN is already making all of the Pool-Runners that it possibly can!

Variable Cells: Allowable Decrease

Next, G.F. Hurley returns to the original problem and considers decreasing one of the coefficients in the objective function. Referring again to Figure 3.2.3, he notices the Sensitivity Report indicates an Allowable Decrease of approximately 7.33 for the coefficient of x_3 in the objective function.

He considers decreasing the coefficient of x_3 by 5 (a value smaller than the Allowable Decrease), 15 (a value greater than the Allowable Decrease) and 7.33 (the Allowable Increase). The corresponding new profit coefficients for each case are 15, 5, and 12.67, respectively.

Figures 3.2.7, 3.2.8, and 3.2.9 show the formulation and Answer Report for each of those decreases in the profit margin of Pool-Runner boards.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	0	104	210			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	35	15			$6,790.00
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	2400	≤	2400
12	Truck Availability	2	2	2	628	≤	700
13	North American Maple Veneers	0	7	0	728	≤	840
14	Chinese Maple Veneers	7	0	7	1470	≤	1470

Figure 3.2.7a: Formulation when the profitability of Pool-Runner boards is $15

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$G$8	Objective Function [Profit ($)] Total Profit	$0.00	$6,790.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [# to make per week] Sporty (x1)	0	0	Contin
$C$6	Decision Values [# to make per week] Fancy (x2)	0	104	Contin
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	0	210	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$11	Shaping Time (minutes)	2400	$E$11<=$G$11	Binding	0
$E$12	Truck Availability	628	$E$12<=$G$12	Not Binding	72
$E$13	North American Maple Veneers	728	$E$13<=$G$13	Not Binding	112
$E$14	Chinese Maple Veneers	1470	$E$14<=$G$14	Binding	0

Figure 3.2.7b: Answer Report when the profitability of Pool-Runner boards is $15

In Figures 3.2.7a and 3.2.7b, the profitability of Pool-Runner boards has been decreased from $20 to $15. This is a decrease of $5, which is smaller than the Allowable Decrease of $7.33. G.F. Hurley notices that the optimal solution is still to make 0 Sporty boards, 104 Fancy boards, and 210 Pool-Runner boards. But the total weekly profit has decreased by $1,050 to $6,790. This is because the profit on each of the Pool-Runner boards made has decreased by $5, and 210 · $5 = $1,050.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	**210**	**90**	**0**			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	35	5			**$6,300.00**
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	**2400**	≤	2400
12	Truck Availability	2	2	2	**600**	≤	700
13	North American Maple Veneers	0	7	0	**630**	≤	840
14	Chinese Maple Veneers	7	0	7	**1470**	≤	1470

Figure 3.2.8a: Formulation when the profitability of Pool-Runner boards is $5

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$G$8	Objective Function [Profit ($)] Total Profit	$0.00	$6,300.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [# to make per week] Sporty (x1)	0	210	Contin
$C$6	Decision Values [# to make per week] Fancy (x2)	0	90	Contin
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$11	Shaping Time (minutes)	2400	$E$11<=$G$11	Binding	0
$E$12	Truck Availability	600	$E$12<=$G$12	Not Binding	100
$E$13	North American Maple Veneers	630	$E$13<=$G$13	Not Binding	210
$E$14	Chinese Maple Veneers	1470	$E$14<=$G$14	Binding	0

Figure 3.2.8b: Answer Report when the profitability of Pool-Runner boards is $5

In Figures 3.2.8a and 3.2.8b, the profitability of Pool-Runner boards has been decreased from $20 to $5. This is a decrease of $15, which is more than the Allowable Decrease. This time, G.F. Hurley notices that the optimal solution has changed from 0 Sporty boards, 104 Fancy boards, and 210 Pool-Runner boards to 210 Sporty boards, 90 Fancy boards, and 0 Pool-Runner boards.

This happened because the profit margin is $15 for Sporty boards, $35 for Fancy boards, and $5 for Pool-Runner boards. In this case, making 210 Sporty boards, 90 Fancy boards, and 0 Pool-Runner boards is more profitable than making 0 Sporty boards, 104 Fancy boards, and 210 Pool-Runner boards. That is, producing 0 Sporty boards, 104 Fancy boards, and 210 Pool-Runner boards generates a profit of:

15(0) + 35(104) + 5(210) = $4690.

On the other hand, producing 210 Sporty boards, 90 Fancy boards, and 0 Pool-Runner boards generates a profit of:

15(210) + 35(90) + 5(0) = $6300.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	210	90	0			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	35	12.66666667			$6,300.00
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	2400	≤	2400
12	Truck Availability	2	2	2	600	≤	700
13	North American Maple Veneers	0	7	0	630	≤	840
14	Chinese Maple Veneers	7	0	7	1470	≤	1470

Figure 3.2.9a: Formulation when profitability of Pool-Runner boards is approximately $12.67

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$G$8	Objective Function [Profit ($)] Total Profit	$0.00	$6,300.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [# to make per week] Sporty (x1)	0	210	Contin
$C$6	Decision Values [# to make per week] Fancy (x2)	0	90	Contin
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$11	Shaping Time (minutes)	2400	$E$11<=$G$11	Binding	0
$E$12	Truck Availability	600	$E$12<=$G$12	Not Binding	100
$E$13	North American Maple Veneers	630	$E$13<=$G$13	Not Binding	210
$E$14	Chinese Maple Veneers	1470	$E$14<=$G$14	Binding	0

Figure 3.2.9b: Answer Report when profitability of Pool-Runner boards is approximately $12.67

In Figures 3.2.9a and 3.2.9b, the profitability of Pool-Runner boards has been decreased from $20 to $12.67. This is a decrease of $7.33, which is the Allowable Decrease. Examining the Answer Report for this case, G.F. Hurley sees that Solver reports 210 Sporty boards, 90 Fancy boards, and 0 Pool-Runner boards as the optimal solution. This yields a total profit of $6,300:

15(210) + 35(90) + 12.67(0) = \$6300.

However, the original optimal solution of 0 Sporty boards, 104 Fancy boards, and 210 Pool-Runner boards also yields a total profit of \$6,300:

15(0) + 35(104) + 12.67(210) = \$6300.

This is another case where the plane representing the objective function coincides with one of the boundaries of the feasible region. Once again, there are infinitely many possible optimal solutions along that boundary.

G.F. Hurley turns his attention to the Allowable Decrease in the objective coefficient of x_2. He notices that Solver reports a value of \$35. Since the profitability of Fancy boards is currently \$35, an Allowable Decrease of \$35 means that no matter how small the profit margin is, as long as Fancy boards generate a positive profit margin, the optimal solution will not change.

Variable Cells: Reduced Cost

Next, G.F. Hurley notices that there is a Reduced Cost of approximately -7.33 listed for Sporty boards (x_1). To see what this means, G.F. Hurley forces the production of one Sporty board by adding the constraint $x_1 = 1$. Figure 3.2.10 shows the new formulation after Solver has found the optimal solution.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	**1**	**103.933333**	**209**			
7							**Total Profit**
8	Objective Function [Profit (\$)]	15	35	20			\$7,832.67
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	**2400**	≤	2400
12	Truck Availability	2	2	2	**627.867**	≤	700
13	North American Maple Veneers	0	7	0	**727.533**	≤	840
14	Chinese Maple Veneers	7	0	7	**1470**	≤	1470
15	Force One Sporty Board	1	0	0	**1**	=	1

Figure 3.2.10: Forcing the production of one Sporty board (x_1)

The new optimal solution is 1 Sporty board, approximately 103.933 Fancy boards, and 209 Pool-Runner boards. (There is nothing wrong with having a non-integer solution since the decision variables are production rates per week, not number of skateboards sold.) This production mix generates a total profit of \$7,832.67 per week. This is a decrease of \$7840 - \$7832.67 = \$7.33 per week. Therefore, **Reduced Cost** refers to the change in the Final Value of the objective function that is caused by increasing a decision variable by one unit.

Q1. SK8MAN has a regular customer who wants to special order 10 Sporty boards. If SK8MAN manufactures those boards, how will that affect profit for that week?

Alternatively, one could think of Reduced Cost as the amount by which the objective function coefficient would have to increase to before it would profitable to make that item. For example, G.F. Hurley notices that the Allowable Increase for Sporty boards is approximately 7.33 and the Reduced Cost for Sporty boards is approximately -7.33. This is not a coincidence! If G.F. Hurley increases the profitability of Sporty boards by more than $7.33, then Sporty boards would be profitable to produce.

To experiment with this idea, G.F. Hurley changes the profitability of Sporty boards to $23. The result is shown in Figure 3.2.11. In this case, it is now profitable to produce Sporty boards.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	**210**	**90**	**0**			
7							**Total Profit**
8	Objective Function [Profit ($)]	23	35	20			**$7,980.00**
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	**2400**	≤	2400
12	Truck Availability	2	2	2	**600**	≤	700
13	North American Maple Veneers	0	7	0	**630**	≤	840
14	Chinese Maple Veneers	7	0	7	**1470**	≤	1470
15							

Figure 3.2.11: Formulation when the profitability of Sporty boards is $23

In a maximization problem, the Reduced Cost value will always be less than or equal to zero. If the Reduced Cost is less than zero, then it is not profitable to make the product. G.F. Hurley considers what it means to have a Reduced Cost value of zero.

The Sensitivity Report in Figure 3.2.3 shows that the Reduced Cost values for the other two decision variables, x_2 and x_3,are both 0. That is because both of those decision variables are already part of the optimal solution. If a decision variable is not part of the optimal solution, its final value is 0. Its Reduced Cost measures the amount the final value of the objective function would be reduced if the value of the decision variable were increased by just 1 unit.

Table 3.2.2 summarizes the impact of changes in the objective function coefficients. The table differentiates between changes within the allowable range and changes outside the allowable range. It also distinguishes between decision variables that are non-zero and those that are zero. If a decision variable is zero, changing the objective function coefficient within the allowable range has no impact on anything including the value of the objective function. Conversely, it is important to note that when the change exceeds the range, the decision variable becomes an non-zero element of the optimal solution.

	Changes to objective function coefficients			
	Decision Variables with non-zero values		**Decision variables with zero values**	
Impact on…	**Within range** of allowable changes	**Outside range** of allowable changes	**Within range** of allowable changes	**Outside range** of allowable changes
Decision Variables	Values do not change	Values change	Values do not change	Values change and decision variable becomes non-zero
Objective Function	Objective function increases or decreases with the change of the coefficient	Increases or decreases with the change of the coefficient and the changes to the decision variables	No change in objective function	Increases or decreases with the change of the coefficient and the changes to the decision variables

Table 3.2.2: Summary of the impact of changes in the objective function coefficients

3.2.4 Solver Sensitivity Report: Constraints

Shadow Price

G.F. Hurley then moves to the Constraints section of the Sensitivity Report (see Figure 3.2.12), where the most useful piece of information is the Shadow Price. The **Shadow Price** tells the effect on the value of the objective function of increasing the resource that is constraining the solution by 1 unit. In other words, the Shadow Price refers to the amount by which the objective function value changes given a 1-unit increase or decrease in one right hand side of a constraint.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$6	Decision Values [# to make per week] Sporty (x1)	0	-7.333333333	15	7.333333333	1E+30
$C$6	Decision Values [# to make per week] Fancy (x2)	104	0	35	40	35
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	210	0	20	1E+30	7.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$11	Shaping Time (minutes)	2400	2.333333333	2400	240	1560
$E$12	Truck Availability	628	0	700	1E+30	72
$E$13	North American Maple Veneers	728	0	840	1E+30	112
$E$14	Chinese Maple Veneers	1470	1.523809524	1470	343.6363636	420

Figure 3.2.12: Sensitivity Report for the 3-variable SK8MAN problem

For example, the shaping time constraint shows a Shadow Price of approximately 2.33. That means if the amount of shaping time were increased by 1 unit, the value of the objective function would increase by 2.33 units.

In the context of the SK8MAN problem, the units of shaping time are minutes, and the units of the objective function are dollars. Suppose the workers agree to work a total of 100 minutes longer each week. Doing so would increase the available shaping time by 100 minutes.

According to the Sensitivity Report, that should increase the value of the objective function for the optimal solution by approximately 100 · \$2.3333 = \$233.33.

Figures 3.2.13a and 3.2.13b show that the objective has increased to \$8,073.33. The new production plan is 0 Sporty boards, approximately 110.67 Fancy boards, and 210 Pool-Runner boards.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	0	110.666667	210			
7							**Total Profit**
8	Objective Function [Profit (\$)]	15	35	20			\$8,073.33
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	2500	≤	2500
12	Truck Availability	2	2	2	641.333	≤	700
13	North American Maple Veneers	0	7	0	774.667	≤	840
14	Chinese Maple Veneers	7	0	7	1470	≤	1470

Figure 3.2.13a: Formulation when available shaping time is increased by 100 minutes

Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$G\$8	Objective Function [Profit (\$)] Total Profit	\$0.00	\$8,073.33

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$6	Decision Values [# to make per week] Sporty (x1)	0	0	Contin
\$C\$6	Decision Values [# to make per week] Fancy (x2)	0	110.6666667	Contin
\$D\$6	Decision Values [# to make per week] Pool-Runner (x3)	0	210	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$E\$11	Shaping Time (minutes)	2500	\$E\$11<=\$G\$11	Binding	0
\$E\$12	Truck Availability	641.3333333	\$E\$12<=\$G\$12	Not Binding	58.66666667
\$E\$13	North American Maple Veneers	774.6666667	\$E\$13<=\$G\$13	Not Binding	65.33333333
\$E\$14	Chinese Maple Veneers	1470	\$E\$14<=\$G\$14	Binding	0

Figure 3.2.13b: Answer Report when available shaping time is increased by 100 minutes

On the other hand, suppose that the available shaping time is reduced by 150 minutes. That reduction should then reduce the final value of the objective function by approximately 150 · \$2.333 = \$350. Figures 3.2.14a and 3.2.14b demonstrate that the objective function has decreased to \$7,490. The new production plan is 0 Sporty boards, 94 Fancy boards, and 210 Pool-Runner boards.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	0	94	210			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	35	20			$7,490.00
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	2250	≤	2250
12	Truck Availability	2	2	2	608	≤	700
13	North American Maple Veneers	0	7	0	658	≤	840
14	Chinese Maple Veneers	7	0	7	1470	≤	1470

Figure 3.2.14a: Formulation when available shaping time is reduced by 150 minutes

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$G$8	Objective Function [Profit ($)] Total Profit	$0.00	$7,490.00

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$6	Decision Values [# to make per week] Sporty (x1)	0	0	Contin
$C$6	Decision Values [# to make per week] Fancy (x2)	0	94	Contin
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	0	210	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$11	Shaping Time (minutes)	2250	$E$11<=$G$11	Binding	0
$E$12	Truck Availability	608	$E$12<=$G$12	Not Binding	92
$E$13	North American Maple Veneers	658	$E$13<=$G$13	Not Binding	182
$E$14	Chinese Maple Veneers	1470	$E$14<=$G$14	Binding	0

Figure 3.2.14b: Answer Report when available shaping time is reduced by 150 minutes

Thus, the Shadow Price shows how much the value of the objective function will increase or decrease for each unit of increase or decrease in the availability of one of the constraining resources.

Constraints: Allowable Increase and Allowable Decrease

Returning once again to the Sensitivity Report in Figure 3.2.12, G.F. Hurley puts his attention towards the columns for an **Allowable Increase** and **Allowable Decrease** for each of the constraints. These refer to increases or decreases in the right hand side of a constraint (i.e., increasing or decreasing the availability of one of the constraining resources, such as shaping time). If an increase or decrease falls within the range determined by the Allowable Increase and Allowable Decrease, then the Shadow Price will remain the same.

For example, from the Sensitivity Report, the Allowable Increase in shaping time is 240 minutes, and the Allowable Decrease is 1,560 minutes. So, if a change in the availability of shaping time falls in the range between an increase of 240 minutes and a decrease of 1,560 minutes, the Shadow Price will stay constant at approximately $2.33.

G.F. Hurley wonders what happens if a change in the available shaping time falls outside this range. He supposes that the shaping time increases by 241 minutes. Since 241 is greater than the Allowable Increase, there should be an effect on the Shadow Price. Figures 3.2.15a and 3.2.15b show the spreadsheet formulation and sensitivity report, respectively, for this change.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty (x_1)**	**Fancy (x_2)**	**Pool-Runner (x_3)**			
6	Decision Values [# to make per week]	0	120	210			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	35	20			$8,400.00
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	2640	≤	2641
12	Truck Availability	2	2	2	660	≤	700
13	North American Maple Veneers	0	7	0	840	≤	840
14	Chinese Maple Veneers	7	0	7	1470	≤	1470

Figure 3.2.15a: Formulation when the shaping time constraint increases by 241 minutes

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$6	Decision Values [# to make per week] Sporty (x1)	0	-5	15	5	1E+30
$C$6	Decision Values [# to make per week] Fancy (x2)	120	0	35	1E+30	35
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	210	0	20	1E+30	5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$11	Shaping Time (minutes)	2640	0	2641	1E+30	1
$E$12	Truck Availability	660	0	700	1E+30	40
$E$13	North American Maple Veneers	840	5	840	0.466666667	840
$E$14	Chinese Maple Veneers	1470	2.857142857	1470	1.75	1470

Figure 3.2.15b: Sensitivity Report when the shaping time constraint increases by 241 minutes

G.F. Hurley notices that the Shadow Price has changed to 0. A Shadow Price of 0 means that there is no value in increasing the availability of installation time any further. Therefore, increasing the availability of a resource beyond the Allowable Increase *decreases* the Shadow Price.

He also notices that increasing the available shaping time changes the optimal solution to 0 Sporty boards, 120 Fancy boards, and 210 Pool-Runner boards per week. Furthermore, the Allowable Increase for shaping time has changed to infinity (1E+30). This means there is no value in increasing the available shaping time any more.

Next, G.F. Hurley investigates what happens if the available shaping time decreases below the Allowable Decrease of 1,560. Suppose he decreases it by 1,561 minutes. The available shaping time becomes 839 minutes. Figures 3.2.16a and 3.2.16b show the spreadsheet formulation and sensitivity report for this change.

	A	B	C	D	E	F	G
1	Chapter 3: Sensitivity Analysis						
2	3.2 SK8MAN, Inc. (3 variables)						
3	Profit Maximization Problem						
4							
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)			
6	Decision Values [# to make per week]	0	0	209.75			
7							**Total Profit**
8	Objective Function [Profit ($)]	15	35	20			$4,195.00
9							
10	**Constraints**						
11	Shaping Time (minutes)	5	15	4	839	≤	839
12	Truck Availability	2	2	2	419.5	≤	700
13	North American Maple Veneers	0	7	0	0	≤	840
14	Chinese Maple Veneers	7	0	7	1468.25	≤	1470

Figure 3.2.16a: Formulation when the shaping time constraint decreases by 1,561 minutes

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$6	Decision Values [# to make per week] Sporty (x1)	0	-10	15	10	1E+30
$C$6	Decision Values [# to make per week] Fancy (x2)	0	-40	35	40	1E+30
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	209.75	0	20	1E+30	8

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$11	Shaping Time (minutes)	839	5	839	1	839
$E$12	Truck Availability	419.5	0	700	1E+30	280.5
$E$13	North American Maple Veneers	0	0	840	1E+30	840
$E$14	Chinese Maple Veneers	1468.25	0	1470	1E+30	1.75

Figure 3.2.16b: Sensitivity Report when the shaping time constraint decreases by 1,561 minutes

G.F. Hurley notices that this change increases the value of the Shadow Price to $5. That is, decreasing the availability of a resource beyond the Allowable Decrease *increases* the Shadow Price. This makes economic sense, because decreasing the availability of a resource, as he did, increases the value per unit of that resource.

Table 3.2.4 summarizes the impact of changes in the right hand side values of the constraints. The table differentiates between changes within the allowable range and changes outside the allowable range. It also distinguishes between binding and non-binding constraints. Unlike objective coefficient changes, RHS changes of binding constraints always change the values of the decision variables. This is true even if the change is within the allowable range. The only difference between within the range and outside the range is whether or not the shadow price changes. This is a complex concept and beyond the scope of this text. If a constraint is non-binding, its shadow price is 0. Thus, any changes within the range have no impact on the decision variables or the objective function. However, when the change exceeds the allowable range for the rhs value, the constraint becomes binding and the shadow price becomes non-zero.

	Changes to right hand side values of constraints			
	Binding Constraints		**Non-binding constraints**	
Impact on…	**Within range** of allowable changes	**Outside range** of allowable changes	**Within range** of allowable changes	**Outside range** of allowable changes
Decision Variables	Values change	Values change	Values do not change	Values change
Objective Function	Objective function change predicted by multiplying shadow price by change in RHS	Objective function change cannot be predicted	No changes	Objective function change cannot be predicted
Shadow price	Stays the same	Changes – Marginal value of resource changes	No changes. Stays at zero.	Shadow price increases from 0 as constraint becomes binding

Table 3.2.4: Summary of the impact of changes in the right hand side values of constraints

3.2.3 Adding a Fourth Product—Is it profitable?

The managers at SK8MAN, Inc. are now considering adding a fourth line of skateboards to their portfolio of products. The EasyRider skateboard will be made from seven North American maple veneers, require 12 minutes of shaping time, and, of course, require 2 trucks. The managers believe that each EasyRider skateboard manufactured will earn $25 profit. They are excited by the prospect of adding a new product to their line, but the key question is whether it will be profitable to do so. Figures 3.2.17 and 3.2.18 display the problem formulation with a fourth decision variable and the optimal solution, as well as the Sensitivity Report.

	A	B	C	D	E	F	G	H
1	Chapter 3: Sensitivity Analysis							
2	3.2 SK8MAN, Inc. (4 variables)							
3	Profit Maximization Problem							
4								
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)	**EasyRider** (x_4)			
6	Decision Values [# to make per week]	**0**	**104**	**210**	**0**			
7								**Total Profit**
8	Objective Function [Profit ($)]	15	35	20	25			**$7,840.00**
9								
10	**Constraints**							
11	Shaping Time (minutes)	5	15	4	12	**2400**	≤	2400
12	Truck Availability	2	2	2	2	**628**	≤	700
13	North American Maple Veneers	0	7	0	7	**728**	≤	840
14	Chinese Maple Veneers	7	0	7	0	**1470**	≤	1470

Figure 3.2.17: Formulation for the 4-decision variable SK8MAN problem

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$6	Decision Values [# to make per week] Sporty (x1)	0	-7.333333333	15	7.333333333	1E+30
$C$6	Decision Values [# to make per week] Fancy (x2)	104	0	35	40	3.75
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	210	0	20	1E+30	7.333333333
$E$6	Decision Values [# to make per week] EasyRider (x4)	0	-3	25	3	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$11	Shaping Time (minutes)	2400	2.333333333	2400	240	1560
$F$12	Truck Availability	628	0	700	1E+30	72
$F$13	North American Maple Veneers	728	0	840	1E+30	112
$F$14	Chinese Maple Veneers	1470	1.523809524	1470	343.6363636	420

Figure 3.2.18: Sensitivity Report for the 4-variable SK8MAN problem

Notice that the optimal solution has not changed, despite the addition of a new product. That means it is not profitable to make the new product. SK8MAN, Inc. will earn more profit by continuing to make only Fancy and Pool-Runner skateboards. Now the question is what, if anything, can be done so that making the new EasyRider boards would be part of SK8MAN's optimal production plan.

To answer that question, G.F. Hurley turns his attention to the Sensitivity Report. Considering the information on the EasyRider board (product x_4), he sees that the Allowable Increase in the objective coefficient is 3. That means that the profitability of EasyRider boards would have to increase by at least $3 (to $28) per board before they would become part of the optimal solution.

The shadow prices on the constraints help explain why the profit margin would need to be at least $28 for each EasyRider skateboard. Each EasyRider board requires 7 North American maple veneers and 2 trucks. The related resource constraints have 0 shadow prices because not all of these resources are currently being used. However, each EasyRider requires 12 minutes of installation. Each minute has a shadow price of $2.333. If G.F. Hurley multiplies 12 by $2.333, he obtains $28. Thus the resources needed to produce an EasyRider board are valued at $28 with the current optimal production plan.

Now, suppose the marketing division at SK8MAN, Inc. has just signed a contract with Allie Loop, the top female skateboarder in the world. She will endorse the new EasyRider board. Taking into consideration the cost of Allie Loop's endorsement contact, the marketing division estimates that the retail price of an EasyRider can be increased by $5. This would then increase the profitability of EasyRider to $30 per board. Since the increase in profitability is larger than the Allowable Increase, this should be enough to make it profitable to produce EasyRider boards.

Figures 3.2.19a and 3.2.19b shows the problem formulation and the Sensitivity Report after increasing the profitability of the EasyRider board (x_4) to $30 per board.

	A	B	C	D	E	F	G	H
1	Chapter 3: Sensitivity Analysis							
2	3.2 SK8MAN, Inc. (4 variables)							
3	Profit Maximization Problem							
4								
5	Decision Variable	**Sporty** (x_1)	**Fancy** (x_2)	**Pool-Runner** (x_3)	**EasyRider** (x_4)			
6	Decision Values [# to make per week]	0	40	210	80			
7								**Total Profit**
8	Objective Function [Profit ($)]	15	35	20	30			$8,000.00
9								
10	**Constraints**							
11	Shaping Time (minutes)	5	15	4	12	2400	≤	2400
12	Truck Availability	2	2	2	2	660	≤	700
13	North American Maple Veneers	0	7	0	7	840	≤	840
14	Chinese Maple Veneers	7	0	7	0	1470	≤	1470

Figure 3.2.19a: Formulation when the profitability of EasyRider boards is $30

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$6	Decision Values [# to make per week] Sporty (x1)	0	-6.666666667	15	6.666666667	1E+30
$C$6	Decision Values [# to make per week] Fancy (x2)	40	0	35	2.5	5
$D$6	Decision Values [# to make per week] Pool-Runner (x3)	210	0	20	1E+30	6.666666667
$E$6	Decision Values [# to make per week] EasyRider (x4)	80	0	30	5	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$11	Shaping Time (minutes)	2400	1.666666667	2400	240	120
$F$12	Truck Availability	660	0	700	1E+30	40
$F$13	North American Maple Veneers	840	1.428571429	840	70	112
$F$14	Chinese Maple Veneers	1470	1.904761905	1470	140	420

Figure 3.2.19b: Sensitivity Report when the profitability of EasyRider boards is $30

When the profitability of the EasyRider board is increased to $30 per board, it becomes profitable to produce 80 of them per week. G.F. Hurley compares this optimal production plan with the optimal production plan before SK8MAN got the Allie Loop endorsement (see Figure 3.2.17). He notices that 210 Pool-Runner boards (x_3) will still be produced, but only 40 Fancy boards will be produced. So, in order to produce 80 EasyRider boards, 64 *fewer* Fancy boards would have to be made.

G.F. Hurley wonders why this is more profitable to produce 64 fewer Fancy boards while producing 80 more EasyRider boards. He considers the profit margins on each of the boards. Making 64 fewer Fancy boards would decrease the total profit by (64)($35) = $2,240. At the same time, making 80 EasyRider boards that were not being made before would increase the total profit by (80)($30) = $2,400. Thus, the total profit is being increased by $2,400 – $2,240 = $160 per week.

Section 3.3: The Pallas Sport Shoe Company

Recall from Chapter 2 that the Pallas Sport Shoe Company manufactures six different lines of sport shoes: High Rise, Max-Riser, Stuff It, Zoom, Sprint, and Rocket. Table 3.3.1 displays the amount of daily profit generated by each pair of shoes for each of these six products. It also lists the amount of time each line of shoes requires for the six steps of production. The last line of the table shows the total amount of time per day available for each of the six production steps. Sue Painter, the production manager of the company would like to determine the daily production rates for each line of shoes that will maximize profit.

	High Rise	**Max-Riser**	**Stuff It**	**Zoom**	**Sprint**	**Rocket**	**Total Time Available (minutes per day)**
Profit ($)	18	23	22	20	18	19	
Stamping (min)	1.25	2	1.5	1.75	1	1.25	420
Upper Finishing (min)	3.5	3.75	5	3	4	4.25	1,260
Insole Stitching (min)	2	3.25	2.75	2.25	3	2.5	840
Molding (min)	5.5	6	7	6.5	8	5	2,100
Sole-to-Upper Joining (min)	7.5	7.25	6	7	6.75	6.5	2,100
Inspecting (min)	2	3	2	3	2	3	840

Table 3.3.1: Profit and production detail per pair for six lines of sport shoes

3.3.1 Problem Formulation

The formulation of the problem is given below.

<u>Decision Variables</u>

Let: x_1 = the daily production rate of High Rise
x_2 = the daily production rate of Max-Riser
x_3 = the daily production rate of Stuff It
x_4 = the daily production rate of Zoom
x_5 = the daily production rate of Sprint
x_6 = the daily production rate of Rocket

<u>Objective Function</u>

Maximize: $z = 18x_1 + 23x_2 + 22x_3 + 20x_4 + 18x_5 + 19x_6$,
where z = the amount of profit Pallas Sport Shoe Company earns per day.

<u>Constraints</u>

Subject to:

$$
\begin{array}{lrcrcrcrcrcrcr}
\text{Stamping (min):} & 1.25x_1 & + & 2x_2 & + & 1.5x_3 & + & 1.75x_4 & + & 1x_5 & + & 1.25x_6 & \leq & 420 \\
\text{Upper Finishing (min):} & 3.5x_1 & + & 3.75x_2 & + & 5x_3 & + & 3x_4 & + & 4x_5 & + & 4.25x_6 & \leq & 1,260 \\
\text{Insole Stitching (min):} & 2x_1 & + & 3.25x_2 & + & 2.75x_3 & + & 2.25x_4 & + & 3x_5 & + & 2.5x_6 & \leq & 840 \\
\text{Molding (min):} & 5.5x_1 & + & 6x_2 & + & 7x_3 & + & 6.5x_4 & + & 8x_5 & + & 5x_6 & \leq & 2,100 \\
\text{Sole-to-Upper Joining (min):} & 7.5x_1 & + & 7.25x_2 & + & 6x_3 & + & 7x_4 & + & 6.75x_5 & + & 6.5x_6 & \leq & 2,100 \\
\text{Inspecting (min):} & 2x_1 & + & 3x_2 & + & 2x_3 & + & 3x_4 & + & 2x_5 & + & 3x_6 & \leq & 840 \\
\text{Non-Negativity (\#):} & x_1 & , & x_2 & , & x_3 & , & x_4 & , & x_5 & , & x_6 & \geq & 0
\end{array}
$$

This formulation as it appears in a spreadsheet is presented in Figure 3.3.1. Solver has been run, and the optimal solution also appears in the spreadsheet.

	A	B	C	D	E	F	G	H	I	J
1	Chapter 3: Sensitivity Analysis									
2	3.3 Pallas Sport Show Company									
3	Profit Maximization									
4										
5	Decision Variable	**High Rise** (x_1)	**Max-Riser** (x_2)	**Stuff It** (x_3)	**Zoom** (x_4)	**Sprint** (x_5)	**Rocket** (x_6)			
6	Decision Values [# to make per day]	**0**	**4.28294434**	**45.12172**	**72.3275**	**104.902**	**89.82118**			
7										**Total Profit**
8	Objective Function [Profit ($)]	18	23	22	20	18	19			$6,132.57
9										
10	**Constraints**							**Used**		**Available**
11	Stamping (minutes)	1.25	2	1.5	1.75	1	1.25	420	≤	420
12	Upper Finishing (minutes)	3.5	3.75	5	3	4	4.25	1260	≤	1260
13	Insole Stitching (minutes)	2	3.25	2.75	2.25	3	2.5	840	≤	840
14	Molding (minutes)	5.5	6	7	6.5	8	5	2100	≤	2100
15	Sole-to-Upper Joining (minutes)	7.5	7.25	6	7	6.75	6.5	2100	≤	2100
16	Inspecting (minutes)	2	3	2	3	2	3	799.34219	≤	840

Figure 3.3.1: Pallas Sport Shoes Spreadsheet Formulation and Optimal Solution

3.3.2 Interpreting the Solution

Q1. Without referring to an Answer or Sensitivity Report, which of the constraints in the spreadsheet in Figure 3.3.1 are binding and which are non-binding? How do you know?

Q2. Similarly, which one of the constraints will show a Shadow Price of zero in the Sensitivity Report, and why does that make sense?

Sue Painter has seen the Answer and Sensitivity Reports. She wonders, “How do I go about implementing this optimal solution?” In order to answer this question, the production manager must understand what the optimal solution means.

Q3. The optimal solution given in the spreadsheet from Figure 3.3.1 lists $x_1 = 0$. What does that mean? What does it mean that $x_2 \approx 4.2829$?

Recalling that the decision variables in the problem were defined as daily production rates, $x_2 \approx 4.2829$means that on most days, 4 Max-Riser shoes will be produced. Then, approximately every fourth day, 5 Max-Riser shoes will be produced. This production plan would yield 4.25 Max-Riser shoes every four days.

Similarly, a daily production rate for $x_3 \approx 45.1217$ means that on most days 45 will be produced, but on about every eighth day, 46 will be produced. This production plan would yield 45.125 Stuff It shoes every eight days.

Q4. How might the production rate of 72.3275 for product x_4 be implemented?

So, in order to implement the optimal production plan, the production manager will have to allocate production resources in such a way that the optimal production rates are achieved.

Figure 3.3.2 shows the Sensitivity Report for the optimal solution to the Pallas Sport Shoe problem. The production manager notices that it reports an Allowable Increase of about \$0.0507 in the coefficient of x_1 in the objective function.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$6	Decision Values [# to make per day] High Rise (x1)	0	-0.05070018	18	0.05070018	1E+30
\$C\$6	Decision Values [# to make per day] Max-Riser (x2)	4.282944345	0	23	0.083039285	0.239247312
\$D\$6	Decision Values [# to make per day] Stuff It (x3)	45.12172352	0	22	0.100842737	1.120253165
\$E\$6	Decision Values [# to make per day] Zoom (x4)	72.32746858	0	20	0.25648415	0.055429065
\$F\$6	Decision Values [# to make per day] Sprint (x5)	104.9019749	0	18	6.260393168	0.529761905
\$G\$6	Decision Values [# to make per day] Rocket (x6)	89.82118492	0	19	0.572583906	0.040500229

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$H\$11	Stamping (minutes) Used	420	5.580179533	420	113.5815474	8.859180036
\$H\$12	Upper Finishing (minutes) Used	1260	1.793895871	1260	15.4507772	111.8007117
\$H\$13	Insole Stitching (minutes) Used	840	0.255655296	840	72.56195965	4.008064516
\$H\$14	Molding (minutes) Used	2100	0.203375224	2100	29.90972919	134.3521595
\$H\$15	Sole-to-Upper Joining (minutes) Used	2100	0.422262118	2100	20.14864865	179.7068966
\$H\$16	Inspecting (minutes) Used	799.3421903	0	840	1E+30	40.65780969

Figure 3.3.2: Sensitivity Report for the Pallas Sport Shoe problem

Q5. Suppose Pallas Shoes was able to increase the profit margin on x_1 to \$18.05. Would this change affect the optimal solution? Why or why not?

Q6. Suppose Pallas Shoes was able to increase the profitability of x_1 to \$18.10. What would be the effect on the optimal solution of this increase?

3.3.3 Using the Sensitivity Report to Make Decisions

Pallas Shoes is considering adding an hour of overtime to one of the workers. Sue Painter must decide to which of the production tasks the overtime should go.

Q7. Using the Sensitivity Report in Figure 3.3.2 to guide the decision, to which of the six production tasks should the extra time be added? Why?

Suppose that the union contract mandates that any overtime work be paid at double the normal rate of $28 per hour.

Q8. Would it be profitable to add to one hour of overtime? If so, how much larger than the cost of the overtime would the increase in profits be? If not, at what hourly pay rate would it be profitable?

Finally, the managers at Pallas Sport Shoes are considering adding another line of shoes. The data for the new Pro-Go model is shown in Table 3.3.2.

	Pro-Go
Profit	$20
Stamping	1.5
Upper Finishing	3.9
Insole Stitching	2.6
Molding	6.3
Sole-to-Upper Joining	6.8
Inspecting	2.5

Table 3.3.2: Profit and production detail per pair of Pro-Go sport shoes

At present, there are no plans to increase the total amount of time available for each of the six steps of production. Figure 3.3.3 contains the new spreadsheet and optimal solution with the information for Pro-Go as decision variable x_7. Figure 3.3.4 shows the Sensitivity Report.

	A	B	C	D	E	F	G	H	I	J	K
1	Chapter 3: Sensitivity Analysis										
2	3.3 Pallas Sport Show Company										
3	Profit Maximization										
4											
5	Decision Variable	**High Rise** (x_1)	**Max-Riser** (x_2)	**Stuff It** (x_3)	**Zoom** (x_4)	**Sprint** (x_5)	**Rocket** (x_6)	**Pro-Go** (x_7)			
6	Decision Values [# to make per day]	**0**	**4.2829443**	**45.1217**	**72.3275**	**104.902**	**89.8212**	**0**			
7											**Total Profit**
8	Objective Function [Profit ($)]	18	23	22	20	18	19	20			$6,132.57
9											
10	**Constraints**								**Used**		**Available**
11	Stamping (minutes)	1.25	2	1.5	1.75	1	1.25	1.5	420	≤	420
12	Upper Finishing (minutes)	3.5	3.75	5	3	4	4.25	3.9	1260	≤	1260
13	Insole Stitching (minutes)	2	3.25	2.75	2.25	3	2.5	2.6	840	≤	840
14	Molding (minutes)	5.5	6	7	6.5	8	5	6.3	2100	≤	2100
15	Sole-to-Upper Joining (minutes)	7.5	7.25	6	7	6.75	6.5	6.8	2100	≤	2100
16	Inspecting (minutes)	2	3	2	3	2	3	2.5	799.34219	≤	840

Figure 3.3.3: Formulation with seven decision variables

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$6	Decision Values [# to make per day] High Rise (x1)	0	-0.05070018	18	0.05070018	1E+30
$C$6	Decision Values [# to make per day] Max-Riser (x2)	4.282944345	0	23	0.083039285	0.239247312
$D$6	Decision Values [# to make per day] Stuff It (x3)	45.12172352	0	22	0.100842737	1.120253165
$E$6	Decision Values [# to make per day] Zoom (x4)	72.32746858	0	20	0.25648415	0.055429065
$F$6	Decision Values [# to make per day] Sprint (x5)	104.9019749	0	18	6.260393168	0.529761905
$G$6	Decision Values [# to make per day] Rocket (x6)	89.82118492	0	19	0.572583906	0.040500229
$H$6	Decision Values [# to make per day] Pro-Go (x7)	0	-0.183813285	20	0.183813285	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$I$11	Stamping (minutes) Used	420	5.580179533	420	113.5815474	8.859180036
$I$12	Upper Finishing (minutes) Used	1260	1.793895871	1260	15.4507772	111.8007117
$I$13	Insole Stitching (minutes) Used	840	0.255655296	840	72.56195965	4.008064516
$I$14	Molding (minutes) Used	2100	0.203375224	2100	29.90972919	134.3521595
$I$15	Sole-to-Upper Joining (minutes) Used	2100	0.422262118	2100	20.14864865	179.7068966
$I$16	Inspecting (minutes) Used	799.3421903	0	840	1E+30	40.65780969

Figure 3.3.4: Sensitivity Report with seven decision variables

Q9. Why was it not profitable to produce the new product?

Q10. How much would its profit margin have to increase to make it profitable enough to produce?

Chapter 3 (Sensitivity Analysis) Homework Questions

1. Recall in Section 2.1.6. The Computer Flips Junior Achievement Company produces four models. Four JA students do the installation work and each of them works 10 hours per week. Another student does the testing work and he also works for 10 hours per week. Market research indicates that the combined sales of Simplex and Multiplex cannot exceed 20 computers per week, and the combined sales of Omniplex and Megaplex cannot exceed 16 computers per week. The table below contains the relevant data.

	Simplex	Omniplex	Multiplex	Megaplex
Profit	200	300	250	400
Installation Time (minutes)	60	120	90	150
Testing Time (minutes)	20	24	24	30

The sensitivity report for the Computer Flips problem is given below. The total profit is $7,000.

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [Computer_Flips.xls]Sheet1

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	x1 Simplex	15	0	200	66.67	26.67
$C$5	x2 Omniplex	0	-20	300	20.00	1E+30
$D$5	x3 Multiplex	0	-20	250	20.00	1E+30
$E$5	x4 Megaplex	10	0	400	100	25.00

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$11	Installation Time	2400	1.67	2400	360	200
$F$12	Testing Time	600	5	600	40	120
$F$13	Market Restriction 1	15	0	20	1E+30	5
$F$14	Market Restriction 2	10	0	16	1E+30	6

a. Computer Flips made its reputation in the 1990s with the launch of Simplex and Omniplex brands. It wants to keep these two classic computers in its product line. If the profit on each Omniplex is increased from $300 to $325, would that be a large enough increase to add Omniplex to the optimal product mix?

b. The five students are not paid per hour. They simply split the weekly profits equally. However, the student in charge of doing testing has expressed am immediate need for extra cash. He requested an opportunity to work a half hour more this week. He agrees to accept an hourly wage of $12/hour but not share in the extra profit. Should the team approve his request?

c. Because of the poor market conditions, the combined demand for Omniplex and Megaplex has decreased from 16 to 8 computers (Market Restriction 2). Computer Flips has decided it cannot sell more than eight of these types of computers this week. How would this affect the optimal solution? Explain.

2. Refer back to Section 2.3.3 SK8MAN Inc. This example includes two decision variables and three constraints as formulated below with a corresponding graphical representation as presented earlier in Figure 2.3.7.

Objective Function

Maximize: $z = 15x_1 + 35x_2$

Constraints

Shaping Time:	$5x_1 + 15x_2 \leq 2400$
Trucks:	$2x_1 + 2x_2 \leq 700$
North American Maple:	$7x_2 \leq 840$
Non-Negativity:	$x_1 \geq 0$ and $x_2 \geq 0$

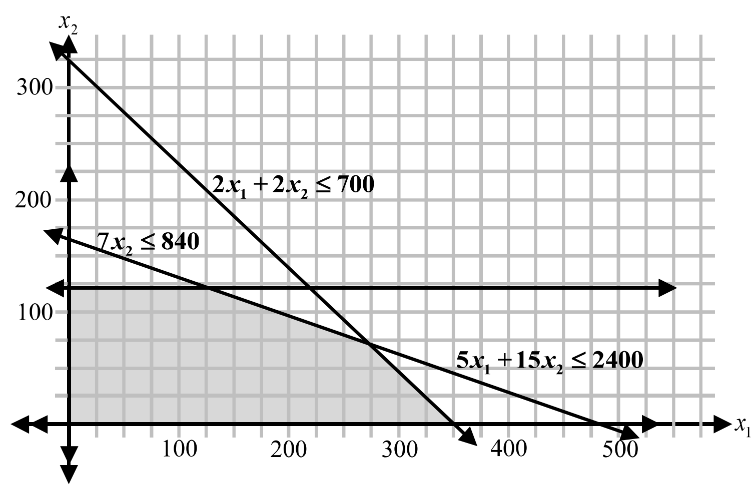


Figure 2.3.7: The SK8man feasible region after adding the North American maple constraint

Answer Report

Cell	Name	Original Value	Final Value
$E$3	Decision variables	0	6550

Cell	Name	Original Value	Final Value
$C$3	Decision variables Sporty	0	285
$D$3	Decision variables Fancy	0	65

Cell	Name	Cell Value	Formula	Status	Slack
$E$6	Shaping Time	2400	$E$6<=$G$6	Binding	0
$E$7	Trucks	700	$E$7<=$G$7	Binding	0
$E$8	North American Maple	455	$E$8<=$G$8	Not Binding	385

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$3	Decision variables Sporty	285	0	15	20	3.33333
$D$3	Decision variables Fancy	65	0	35	10	20

Constraints

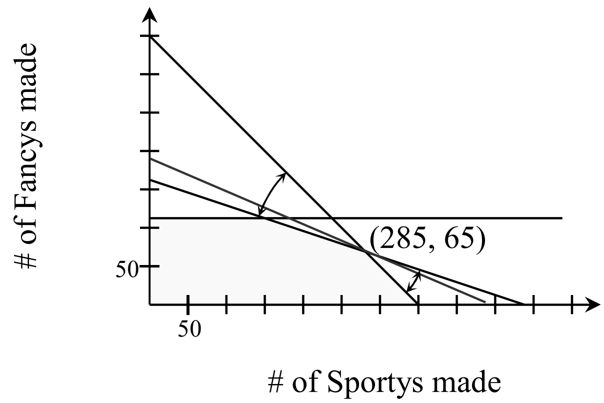
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$6	Shaping Time	2400	2	2400	550	650
$E$7	Trucks	700	2.5	700	260	220
$E$8	North American Maple	455	0	840	1E+30	385

a. There is a rumor that a competitor will soon start selling a skateboard directly comparable to Fancy. This will cause the profit margin to decline by $6 to $29. If this rumor comes true, how would that impact the optimal solution and total profit?

b. There is a rumor that a competitor will soon start selling a skateboard directly comparable to Sporty. This will cause the profit margin to decline by $3.50 to $11.50. If this rumor comes true, how would that impact the optimal solution and total profit?

c. SK8man is considering enhancements to their Sporty product that will increase the profit margin by $4 to $19. How would that impact the optimal solution and total profit?

d. Explain graphically what happens to the objective function when the profit margin on Fancy increases by $10 to a $45. What information in the above reports suggests this would happen?



e. Describe a corresponding change in profit for the Sporty product that would have the same impact.

f. In general, maintenance on the shaping equipment is done before work starts each day. Occasionally workers must turn off the equipment and realign critical components. This takes 30 minutes. What is the impact on the total profit of this loss of production time? How would this change the optimal production plan?

g. Management is considering extending the use of shaping equipment by an hour. How would this affect total profit?

h. The local supplier of trucks is willing to provide 100 extra trucks per shipment. How would this impact the optimal production plan?

i. Assume now that these extra trucks come at a premium price that adds $3 to the cost. Should management be willing to purchase these extra trucks? What if the premium price were only $1.50 what would be the decision? What information in the sensitivity report helps resolve these questions?

j. The supplier of North American Maple is prepared to offer an additional 100 veneers at a discount price. Should management purchase these extra veneers? Explain.

3. Recall from Chapter 2 homework that Anderson Cell Phone Company produces smart phones and standard phones. There are 10 workers available who are each available to work 7 hours per day on assembly. The table below gives the assembly times (in minutes) required to assemble each type of phone and their associated profits. 2000 LCD screens are available per day.

	Smart Phone	Standard Phone
Profit	$40	$30
Assembly Time	2.5	1.5

Below are the Answer and the Sensitivity Reports for the problem.

Target Cell (Max)

Cell	*Name*	*Original Value*	*Final Value*
$D$2	*OBJ. FNC.*	*$0*	*$72,000*

Adjustable Cells

Cell	*Name*	*Original Value*	*Final Value*
$B$3	*D.V. Smart*	*0*	*1200*
$C$3	*D.V. Standard*	*0*	*800*

Constraints

Cell	*Name*	*Cell Value*	*Formula*	*Status*	*Slack*
$D$5	*Assembly Time*	*4200*	*$D$5<=$F$5*	*Binding*	*0*
$D$6	*Screen*	*2000*	*$D$6<=$F$6*	*Binding*	*0*

Adjustable Cells

Cell	*Name*	*Final Value*	*Reduced Cost*	*Objective Coefficient*	*Allowable Increase*	*Allowable Decrease*
$B$3	*D.V. Smart*	*1200*	*0*	*40*	*10*	*10*
$C$3	*D.V. Standard*	*800*	*0*	*30*	*10*	*6*

Constraints

Cell	*Name*	*Final Value*	*Shadow Price*	*Constraint R.H. Side*	*Allowable Increase*	*Allowable Decrease*
$D$5	*Assembly Time*	*4200*	*10*	*4200*	*800*	*1200*
$D$6	*Screen*	*2000*	*15*	*2000*	*800*	*320*

a. What are the binding constraints?

b. What would be the daily profit if one assembly worker did not come to work due to illness and there were only 9 workers available?

c. Check your answer to part (c) by reusing Solver to determine the optimal solution with 9 workers.

4. Because of intense market competition, the selling price for standard phones has decreased by $5 with a similar decrease in the profit for a standard phone.

   a. Graph the feasible region and show the line of constant profit passing through the optimal corner point. Does the optimal solution change? What is the objective function value?

   b. What if the price decrease is $6? $7? What are the new values of the objective function?

   c. What information in the sensitivity report helps explain why a $5 and $7 decrease do not similarly impact the optimal production plan?

5. Below is the sensitivity report for the GA Sports Soccer Ball homework problem in Chapter 2. The thermal molding machine is down for 20 minutes.

GA Soccer Ball Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	D.V. Professional	15	0	15	5	15
$C$3	D.V. Practice	30	0	10	1E+30	2.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Cutting Time	420	1.071	420	14	210
$D$6	Hand-stitching	15	0	16	1E+30	1
$D$7	Thermal Molding	360	0.208	360	360	24

   a. The thermal molding machine needs emergency repairs and could not be used for 20 minutes. How much did it cost to the company in lost profit?

   b. The company is considering hiring more stitchers. Is this a good idea?

   c. The net profit per ball already includes the salary of workers during the regular day. However, the cutter informed management that he was available to work one hour overtime with overtime pay that is an extra $8per hour more than regular pay. Should management increase the cutter's hours? Justify your answer.

d. GA is concerned that increasing competition may force them to cut their profit margins on practice balls by $2 per ball. Would this cause them to change their production plan? How much would their profits decrease?

6. Consider the four product version of the Family Cow dairy in chapter 2. The owner of the dairy can make and sell cream, butter, plain yogurt and cheese. The prices for these products are $7/quart, $12/lb, $3.5/lb, and $5.5/lb, respectively.

   a. Create the Answer and Sensitivity Reports for the problem.

   b. The farmer's nephew is considering helping out after school. He is available two hours a day but wants to be paid $10 per hour for his time? Would it be worth hiring him? If so what would be the net impact on the daily profit?

   c. The farmer was surprised that the optimal plan did not include cream. He wonders if a $0.25 increase in the sale price for cream would change his production plan. Would the plan be affected by this price increase?

   d. The farmer's best customer insists she needs two quarts of cream tomorrow. What will be the impact on his total profit? Rerun Solver to determine his new optimal plan for that day.

   e. There is a holiday coming up. The local store believes it can sell 10 pounds of butter per day. What will be the new objective value?

7. In Chapter 2 homework, Katia considered investing her money in a bond fund and a domestic stock fund.

Sensitivity Analysis

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	D.V. bond	133333.33	0	0.08	0.07	0.0425
$C$3	D.V. stock	66666.67	0	0.15	0.17	0.07

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Max Stock Investment	66666.67	0	75000	1E+30	8333.333
$D$6	Max Risk	20000	0.467	20000	1250	10000
$D$7	Max Investment	200000	0.057	200000	200000	25000

   a. What are the binding and non-binding constraints for her decision?

b. Why is the shadow price for the maximum stock investment constraint 0?

c. Which would have a greater impact on total revenue, adding $1,000 to the total investment or increasing the allowable risk by $1,000?

d. Increase the total investment by $1,000 and describe how the optimal investment strategy changed.

8. In chapter 2 Katia also considered investing in an international stock fund. She is concerned that the forecasted return is overly optimistic.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	D.V. bond	150000	0	0.08	0.14	0.036
$C$3	D.V. domestic stock	0	-0.035	0.15	0.035	1E+30
$D$3	D.V. international stoc	50000	0	0.22	0.18	0.046666667

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$5	Max Stock Investment	$50,000	$0.00	75000	1E+30	25000
$E$6	Max Risk	$20,000	$0.70	20000	5000	10000
$E$7	Max Investment	$200,000	$0.05	200000	200000	100000

a. Would the optimal investment strategy change if the return on investment for the international fund were only 20%.

b. What change to the projected annual return for the domestic stock will result in it being included in the optimal investment plan?

c. Katia is considering setting aside $5,000 to pay off some bills. How much revenue would she lose as a result?

d. How much increased revenue would she gain if she were willing to accept a $1,000 increase in risk? Rerun Solver to determine the new optimal solution and show how the new optimal investment plan confirms your answer.

9. Remember John Farmer from Chapter 2's set of homework problems? He would like to help his father find the optimal crop mix to plant in the 640-acre farm under cultivation. Mr. Farmer had budgeted $60,000 to cover all expenses. Use the information given in the table to answer the sensitivity questions.

	Corn	Soybean	Wheat
Price/bushel	$2.90	$7.85	$3.75
Yield/acre	155.8 bushels	41.4 bushels	60 bushels
Seed cost/acre	$45.50	$34	$24
Fertilizer cost/acre	$88.10	$37.80	$69.25
Fuel cost/acre	$24.40	$10	$40
Worker cost/acre	$13	$9.50	$9

The sensitivity report for the crop mix problem is given below:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$17	Corn	19.67	0	280.82	156.87	47.13
$C$17	Soybean	620.33	0	233.69	47.13	83.76
$D$17	Wheat	0	-181.07	82.75	181.07	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$19	Land	640	179.70	640	17.17	289.12
$F$20	Budget	60000	0.59	60000	49440	1568

a. A farmers' association announced it will grant low interest credit to local farmers. The interest rate is 7%. Mr. Farmer is considering increasing his budget by borrowing $10,000. How much will the net income increase with the new budget?

b. In order to encourage corn production of corn, the USDA will provide a subsidy that will increase the corn price per bushel to $3.50. Does it affect Mr. Farmer's decision?

c. Mr. Farmer is considering diversifying his crop by planting 10 acres of wheat. How would this impact his total profit?

d. Mr. Farmer's daughter wants her father to delay planting wheat until the price increases to the point that wheat is part of the optimal solution. At what price should Mr. Farmer consider producing wheat?

e. The owner of the neighboring farm suggested leasing his small farm to Mr. Farmer. He is asking for $1000 for this 10 acre farm. Is this a good offer for Mr. Farmer?

10. In chapter 2 homework, Elegant Fragrances, Ltd developed an optimal solution for its 4 product lineup.

    a. The company is concerned about frequent changes in the market for their perfumes. These changes can cause prices to drop by 10% or even 20%. Would a decline of 10% in any perfume price cause the optimal solution to change? What about a 20% decline?

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$E$3	Decision X1	14285.71	0.00	324.25	1E+30	59.47
$F$3	Decision X2	5595.24	0.00	191.52	103.12	41.19
$G$3	Decision X3	0.00	-76.46	436.3	76.46	1E+30
$H$3	Decision X4	9880.95	0.00	210.46	57.67	70.20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$J$6	Mango Pulp 4 Perfumes	1400.0	2148.4	1400	51.43	338.78
$J$7	Tea Leaves 4 Perfumes	1600.0	1201.4	1600	257.14	191.84
$J$8	Juniper Berry 4 Perfumes	1000.0	2853.2	1000	470.00	90.00
$J$9	White Rose 4 Perfumes	1435.7	0.0	1500	1E+30	64.29

    b. The company is considering buying more raw materials. Which raw materials does it need to buy? Which of these will have the greatest impact per pound? How can a shadow price per pound of raw material be more than $1,000 when the profit margins are always less than $500?

    c. The optimal solution included only three of the four products. The optimal does not include Evergreen. However, management wants to offer a full range of products. It is thinking of requiring a minimum production of 1,000 lbs. of each perfume. They know this will reduce profits but wonder if the reduction in profit will be more than 2%. Explain why the impact of this requirement is going to be small relative to the total profit which is more than $8.45 million. Rerun the model with this requirement and describe the impact on production

Chapter 3 Summary

What have we learned?

The process of **sensitivity analysis** is used to explore the robustness of the optimal solution to a linear programming problem. Oftentimes there is uncertainty or variability in the parameters of a problem. Sensitivity analysis allows the decision maker to understand how sensitive the optimal solution is to changes in these parameters. Knowing the implications of changes to parameters also allows for managers to make better decisions about how operations can be improved.

1. Solve the problem and generate reports.
   - Set up spreadsheet formulation of problem.
   - Set up Solver Parameters and Options.
   - Solve and generate Answer and Sensitivity Reports.

2. Interpret the Answer Report.
   - Know what information is contained in the Answer Report.
   - Know how the same information is represented in the solved spreadsheet.

3. Interpret the Sensitivity Report.
   - Know what information is contained in the Sensitivity Report.
   - Investigate effects of changing objective function coefficients.
   - Investigate effects of changing constraint right hand sides.
   - Understand what changes affect optimal solution.
   - Understand what changes affect the optimal value of the objective function.

Terms

Answer Report A report generated by Solver after it has found the optimal solution. This report summarizes the optimal solution, the optimal value of the objective function, and the status of the constraints.

Binding When a constraint models the consumption of a resource, it is binding if the solution to the problem uses all of the resource that is available. This is the case when the left hand side and the right hand side of a constraint are equal.

Parameters The data that define a problem. These include objective function coefficients, coefficients within constraints, and constraint right hand side values.

Sensitivity Analysis The process of exploring how changing the parameters of a linear programming problem affects the optimal solution and optimal value of the objective function.

Sensitivity Report A report generated by Solver after it has found the optimal solution. This report provides information to predict how the optimal solution or the optimal value of the objective function will change in response to varying specific parameters of the problem.

	A	B	C	D	E	F	G
1	**2.2 SK8MAN, Inc.**						
2	**LP - Max**						
3		Sporty	Fancy	Pool-Runner			
4	**Decision Variables**	x1	x2	x3			
5	Decision Variable Values (#/week)	**0**	**104**	**210**			
6					**Total Profit**		
7	**Objective Function** [Profit ($/week)]	15	35	20	**7840**		
8							
9	**Constraints**						
10	Shaping time (min/week)	5	15	4	**2400**	≤	2400
11	Truck availability (#/week)	2	2	2	**628**	≤	700
12	North American maple veneers (#/week)	0	7	0	**728**	≤	840
13	Chinese maple veneers (#/week)	7	0	7	**1470**	≤	1470

Spreadsheet formulation showing optimal solution

	A	B	C	D	E	F	G
6	Target Cell (Max)			E	C		
7	D	Cell	Name	Original Value	Final Value		
8		$E$7	Objective Function [Profit ($/week)] Total Profit	0	7840		
9							
10							
11	Adjustable Cells			E	C		
12		Cell	Name	Original Value	Final Value		
13		$B$5	Decision Variable Values (#/week) x1	0	0		
14		$C$5	Decision Variable Values (#/week) x2	0	104		
15		$D$5	Decision Variable Values (#/week) x3	0	210		
16							
17		B					
18	Constraints			A		G	F
19		Cell	Name	Cell Value	Formula	Status	Slack
20		$E$10	Shaping time (min/week) Total Profit	2400	$E$10<=$G$10	Binding	0
21		$E$11	Truck availability (#/week) Total Profit	628	$E$11<=$G$11	Not Binding	72
22		$E$12	North American maple veneers (#/week) Total Profit	728	$E$12<=$G$12	Not Binding	112
23		$E$13	Chinese maple veneers (#/week) Total Profit	1470	$E$13<=$G$13	Binding	0

Answer report

A: Cell Value — Values in this column show the total for the left hand side of each constraint based on the optimal solution.

B: Constraints — The section of the Answer Report dealing with the system of constraints.

C: Final Value — The value that appeared in the indicated cell after the "Solve" button was pushed.

D: Objective/Target Cell — The section of the Answer Report dealing with the objective function.

E: Original Value — The value that appeared in the indicated cell before the "Solve" button was pushed.

F: Slack — The difference between the right hand side and the left hand side values of a constraint.

G: Status — Shows whether a constraint is binding or not binding.

H: Variable/Adjustable Cells — The section of the Answer Report dealing with the decision variable values.

	A	B	C	D	E	F	G	H
6	Adjustable Cells				H	G	D	B
7	J			Final	Reduced	Objective	Allowable	Allowable
8		Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9		$B$5	Decision Variable Values (#/week) x1	0	-7.33333333	15	7.33333333	1E+30
10		$C$5	Decision Variable Values (#/week) x2	104	0	35	40	35
11		$D$5	Decision Variable Values (#/week) x3	210	0	20	1E+30	7.33333333
12	F							
13	Constraints				I	E	C	A
14				Final	Shadow	Constraint	Allowable	Allowable
15		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
16		$E$10	Shaping time (min/week) Total Profit	2400	2.333333333	2400	240	1560
17		$E$11	Truck availability (#/week) Total Profit	628	0	700	1E+30	72
18		$E$12	North American maple veneers (#/week) Total Profit	728	0	840	1E+30	112
19		$E$13	Chinese maple veneers (#/week) Total Profit	1470	1.523809524	1470	343.636364	420

Sensitivity report

A: Allowable Decrease (Constraints) — The amount the constraint right hand side can decrease without affecting the shadow price.

B: Allowable Decrease (Variable/Adjustable) — The amount the objective function coefficient can decrease without affecting the optimal solution.

C: Allowable Increase (Constraints) — The amount the constraint right hand side can increase without affecting the shadow price.

D: Allowable Increase (Variable/Adjustable) — The amount the objective function coefficient can increase without affecting the optimal solution.

E: Constraint R.H. Side — When a constraint models the consumption of a resource, the right hand side value shows the amount of that resource that is available.

F: Constraints — The section of the Sensitivity Report dealing with the system of constraints.

G: Objective Coefficient — Lists the objective function coefficients for each decision variable.

H: Reduced Cost — The amount the optimal value of the objective function will change in order to increase a decision variable value from zero to one.

I: Shadow Price — The amount of change in the optimal value of the objective function if the right hand side of the constraint were increased by one unit.

J: Variable/Adjustable Cells — The section of the Sensitivity Report dealing with the decision variable values.

Chapter 3 (Sensitivity Analysis) Objectives

You should be able to:

- Enter the problem formulation into Excel
- Set up Solver Parameters and Options
- Interpret the optimal solution in the context of the problem
- Analyze the Answer Report
- Analyze the Sensitivity Report
- Know the type of information contained in the sections of the Answer and Sensitivity Reports
- Differentiate between Allowable Increase/Decrease in adjustable cells and in constraints
- Explain meaning of Shadow Price being positive, negative, or zero
- Understand meaning of Reduced Cost

Chapter 3 Study Guide

1. What information have we used from the Answer Reports?

2. Write a definition for each in your own words.
   a. Binding constraint

   b. Non-binding constraint

3. What is the "Final Value" on an Answer Report?

4. What is slack? What does the cell value tell us?

5. What information have we used from the Sensitivity Reports?

6. For the decision variables, what do the allowable increase and decrease tell us about the variables?

7. What does the reduced cost tell us?

8. Complete each statement describing what happens to the objective function:
   a. If the shadow price for a constraint is 0, then

   b. If the shadow price for a constraint is 100, then

9. How does "Constraint R. H. Side" relate to the word problem?

10. What information is listed in the final value column?

11. Which report would you use to find the final value of the objective function?

12. In 5 or more COMPLETE, GRAMMATICALLY CORRECT sentences compare and contrast the answer report and sensitivity report. Tell how they can be used to analyze problems. **<u>Use at least one example</u>** of how we have used them with the problems done in class.