

# CHAPTER 2:

Optimize Product Mix: Profit Maximization with Linear Programming





## Section 2.0: Mathematical Programming

The next five chapters in the text focus on mathematical programming. The father of mathematical programming is George Dantzig. Between 1947 and 1949, Dantzig developed the basic concepts used for framing and solving linear programming problems. During WWII, he worked on developing various plans which the military called “programs.” After the war he was challenged to find an efficient way to develop and solve these programs.

Dantzig recognized that these programs could be formulated as a system of linear inequalities. Next, he introduced the concept of a goal. At that time, goals usually meant rules of thumb for carrying out a goal. For example, a navy admiral might have said, “Our goal is to win the war, and we can do that by building more battleships.” Dantzig was the first to express the selection of a plan to reach a goal as a mathematical function. Today it is called the *objective function*.

All of this work would not have had much practical value without a way to solve the problem. Dantzig found an efficient method called the simplex method. This mathematical technique finds the optimal solution to a set of linear inequalities that maximizes (profit) or minimizes (cost) an objective function.

Economists were excited by these developments. Several attended an early conference on linear programming and the simplex method called “Activity Analysis of Production and Allocation.” Some of them later won Nobel prizes in economics for their work. They were able to model fundamental economic principles using linear programming.

The first problem Dantzig solved was a minimum cost diet problem. The problem involved the solution of nine inequalities (nutrition requirements) with seventy-seven decision variables (sources of nutrition). The National Bureau of Standards supervised the solution process. It took the equivalent of one man working 120 days using a hand-operated desk calculator to solve the problem. Nowadays, a standard personal computer could solve this problem in less than one second. Excel spreadsheet software includes a standard “add-in” called “Solver”, a tool for solving linear programming problems.

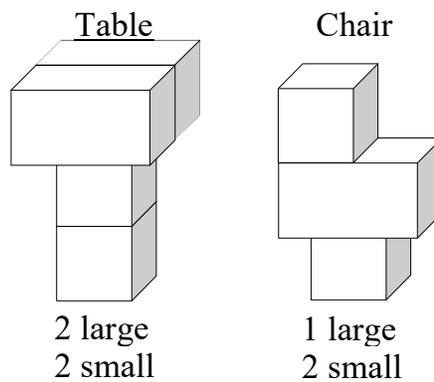
Mainframe computers became available in the 1950s and grew more and more powerful. This allowed many industries, such as the petroleum and chemical industries, to use the simplex method to solve practical problems. The field of linear programming grew very fast. This led to the development of non-linear programming, in which inequalities and/or the objective function are not linear functions. Another extension is called integer programming, in which the variables can only have integer values. Together, linear, non-linear and integer programming are called **mathematical programming**.

## 2.1 An Introductory Problem: Lego Furniture

In order to get a feel for mathematical programming, this chapter begins with a problem that has a concrete model. This model can be built from Lego pieces. When a mathematical model of a real world situation is constructed in symbolic form, it is often helpful to construct a physical or visual model at the same time. The role of the latter model is to help the model builder to understand the real-world situation as well as its mathematical model.

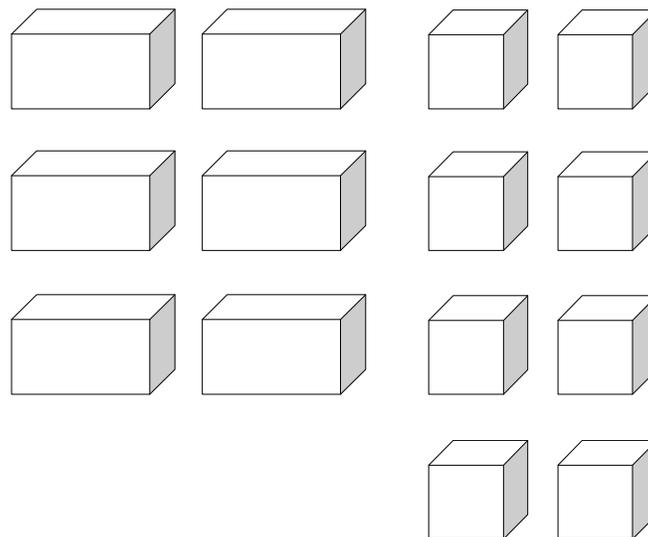
### The Problem

A certain furniture company makes only two products: tables and chairs. The manufacturing of tables and chairs can be modeled using Lego pieces. To make a table requires two large and two small pieces, and a chair requires one large and two small pieces. Figure 2.1.1 shows a table and a chair made from Legos.



**Figure 2.1.1:** A Lego table and chair

If the resources needed to build tables and chairs were unlimited, the company would just manufacture as many of each as it thought it could sell. In the real world, however, resources are not unlimited. Suppose that the company can only obtain six large and eight small pieces per day. Figure 2.1.2 shows these limited resources.



**Figure 2.1.2:** The furniture company's limited resources

The profit from each table is \$16, and the profit from each chair is \$10. The production manager wants to find the rate of production of tables and chairs per day that earns the most profit.

**Production rate** refers to the number of tables and chairs this company can produce per day.

- Q1. What do you think the production rates should be in order to generate the most profit?
- Q2. Does the number of table and chairs produced each day have to be an integer value?
- Q3. Using only eight small and six large Legos, build a physical model of this problem. If Legos are unavailable, draw pictures to explore some possibilities. Create several combinations of tables and chairs this company could make using your model.

### Solving the Problem

There are many possible product mixes this company could make. A **product mix** is a combination of each product being manufactured. The various product mixes could be explored using the Lego model.

First, the company could begin by making as many tables as possible since the profit from a table is much greater than the profit from a chair. Each table requires two large pieces and two small pieces. There are only six large and eight small pieces available. Therefore, only three tables can be built. This generates  $3(\$16) = \$48$  profit. There are two small pieces left over, but nothing can be built from them. Thus, \$48 is the total profit if three tables (and no chairs) are built.

Three tables and zero chairs was one possible product mix. There could be other production rates that generate more profit.

No more than three tables could be made due to the limited resources available, and making three tables yielded a profit of \$48. Now, suppose two tables are made. Manufacturing two tables uses four large and four small pieces. Now there are two large and four small pieces left over. These are just enough resources to build two chairs. The profit on two tables and two chairs would be  $2(\$16) + 2(\$10) = \$52$ . This is more profit than building three tables. However, the production manager wonders, “Is \$52 the greatest profit possible? Is there another product mix that could generate more profit?”

- Q4. In a Table 2.1.1, record other combinations of tables and chairs the company could produce. For each combination, write the production rate of tables, the production rate of chairs, and the profit for each possibility.

Production Rate of Tables	Production Rate of Chairs	Total Profit

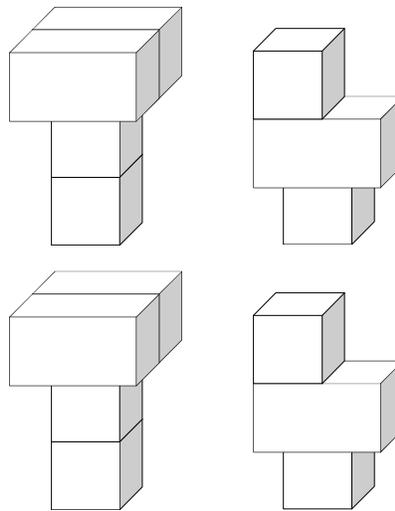
**Table 2.1.1:** Exploring the total profit for each combination of tables and chairs

Q5. Which production rates generate the most profit?

Q6. Did any product mix yield a profit greater than \$52?

It is impossible to find the total profit for every product mix because there are infinitely many possibilities. However, most likely no one in the class found a profit greater than \$52. In the next section, you will learn how to know for certain you found the product mix with the greatest profit.

Notice that in Table 2.1.1 you used a set of similar equations to compute the profit for each possibility. These equations are the basis for the **objective function**. The two production rates *varied* across each possible product mix, and exploring these variations allows a *decision* about production to be made. Therefore, the production rates for tables and chairs are known as the **decision variables** for this problem.



**Figure 2.1.3:** Two tables and two chairs yield the most profit

Because the profit has been optimized, the solution in Figure 2.1.3 is called the **optimal solution**. Besides the optimal solution, there are many other possible solutions. Although they are not optimal, each possible solution is still a **feasible** solution. Building four tables is an example of an **infeasible** solution.

Q7. Why is building four tables an example of an infeasible solution?

Q8. Give another example of an infeasible solution.

### Stepping Beyond the Current Optimal Solution

Operations researchers understand that there is more to their work than merely finding solutions to problems. Once a solution is found, it must be interpreted. One sort of interpretation is called **sensitivity analysis**. Sensitivity analysis involves exploring how sensitive the solution is to changes in the parameters of the problem. For example, in the Lego problem above, one of the parameters of the problem is the availability of large pieces.

- Q9. Would it make a difference if *seven* large pieces were available instead of six (there are still eight small pieces)? If so, what is the new optimal solution, and how much profit does it generate?
- Q10. Would it make a difference if *nine* small pieces were available instead of eight (there are still six large pieces)? If so, what is the new optimal solution, and how much profit does it generate?
- Q11. Would it make a difference if *seven* large pieces and *nine* small pieces were available? If so, what is the new optimal solution, and how much profit does it generate?

### Growing the Problem

Suppose now that the furniture company has decided to dramatically expand production. Now it is able to obtain 27 small and 18 large Lego pieces per day. The profit on tables and chairs remains the same.

- Q12. Use Table 2.1.2 to complete the following:
- Make a suggestion for the number of tables and chairs to produce in this expanded problem.
  - Calculate the combined total number of small Lego pieces and large Lego pieces that are used in this suggestion. Be sure that both of the totals do not exceed the available number of Lego pieces.
  - Calculate the total profit for the suggested production rate.
  - Repeat these steps four more times, for a total of five suggestions. With each new suggestion, try to produce more profit than the suggestion before.

	Production Rate of Tables	Production Rate of Chairs	Number of Small Lego Pieces Used	Number of Large Lego Pieces Used	Total Profit
Suggestion 1					
Suggestion 2					
Suggestion 3					
Suggestion 4					
Suggestion 5					

**Table 2.1.2:** Exploring the total profit for different combinations of tables and chairs

- Q13. What difficulties did you experience when developing the feasible production plans in Q12?

## Section 2.2: Computer Flips, a Junior Achievement Company

Junior Achievement (JA) is an educational program available worldwide. JA uses hands-on experiences to help young people understand the economics of life. In partnership with businesses and educators, JA brings the real world to students. The JA Company Program provides basic economic education for high school students by using support and guidance of volunteer consultants from the local business community. By organizing and operating an actual business, students learn how businesses function. They also learn about the structure of the free enterprise system and the benefits it provides.

Gates Williams is the production manager for Computer Flips, a Junior Achievement company. Computer Flips purchases a basic computer at wholesale prices and then adds a display, extra memory cards, extra USB ports, or a CD-ROM or DVD-ROM drive. The company also purchases these extra components at wholesale prices. The computers, with the added features, are then resold at retail prices.

Computer Flips produces two models: Simplex and Omniplex. The profit on each Simplex is \$200, and on each Omniplex, the profit is \$300. The Simplex model has fewer add-ons, so it requires only 60 minutes of installation time. The Omniplex has more add-ons and requires 120 minutes of installation time. Five JA students do all of the installation work. Each of them works 8 hours per week. Gates Williams must decide the rate of production per week of each computer model in order to maximize the company's weekly profit.

To make decisions such as the one Gates Williams faces, operations researchers use a technique known as **linear programming**. Answering the following questions will help you understand this technique.

### 2.2.1 Exploring the Problem

One way to approach the problem is to make some guesses and test the profit generated by each guess. For example, suppose Gates Williams decides the company should make 20 of each model.

- Q1. How much profit would be generated?
- Q2. Is there enough installation time available to make that number of each model?
- Q3. Answer the same two questions if Gates Williams decides to make:
  - a. 10 Simplex computers and 30 Omniplex computers
  - b. 30 Simplex and 10 Omniplex
- Q4. Can you find a product mix for which there is enough installation time?
- Q5. How much profit do the production rates you found generate for the company?

### 2.2.2 Generalizing the Problem

Sometimes it is helpful to visualize things. The numbers, variables, and their relationships in a problem can be represented by a graph. Before graphing the Computer Flips problem, you must translate the information in the problem into mathematical statements—equations or inequalities.

First, let

- $x_1$  represent the weekly production rate of Simplex computers and
- $x_2$  represent the weekly production rate of Omniplex computers.

The variables  $x_1$  and  $x_2$  are called decision variables because Gates Williams uses them to help make his decision. Mr. Williams's goal is to make as much money as possible. He does this by selling as many computers as he is able. Therefore, Mr. Williams can calculate his weekly profit ( $z$ ) as a function of  $x_1$  and  $x_2$ . Because the objective is to maximize profit, the profit function is called the objective function.

- Q6. Write an equation for the profit ( $z$ ) the company would earn in a week. [Hint: Look back at Section 2.1.1 and see how you calculated profit for 20 Simplex and 20 Omniplex computers.]
- Q7. Write a mathematical statement in terms of  $x_1$  and  $x_2$  that describes the relationship between the installation time required and the installation time available. [Hint: Look back at Section 2.1.1 and see how you determined if there was enough installation time to produce 20 Simplex and 20 Omniplex computers.]
- Q8. Can Computer Flips produce a negative number of either model?
- Q9. Write two mathematical inequalities that represent the answer to the previous question.

The mathematical statements created in this section will be used to find the optimal solution in the following sections.

### 2.2.3 A Visual Approach

At this point, it should be clear that Gates Williams cannot decide to make any number of each model he chooses because there is only a certain amount of installation time available each week. That is, the available installation time *constrains* the number of Simplex and Omniplex computers that can be made each week. The inequality that captures this relationship (from Q7) is called a **constraint**. The other two inequalities (from Q9) express the fact that the decision variables in this problem cannot be negative. Thus, they are called **non-negativity constraints**.

These constraints can be graphed on a coordinate plane. This graph gives a visual representation of the possible production rates for each computer model.

- Q10. On the same coordinate axes, graph each of the three inequalities you wrote in the previous section (one from Q7 and one from Q9). For uniformity, place  $x_1$  on the horizontal axis and  $x_2$  on the vertical axis.
- Q11. Give one point that satisfies all three inequalities.
- Q12. Where are all of the points that satisfy all three inequalities?
- Q13. What is the connection between the points identified in the previous question and the Simplex and Omniplex computers?

The points that satisfy each of the constraint inequalities represent a mix of Simplex and Omniplex computers that could be produced each week. Recall that this region of the coordinate system is called the feasible region, because those points represent feasible production mixes.

- Q14. Choose any point in the feasible region, and compute the weekly profit that would be generated by producing that mix of Simplex and Omniplex computers.
- Q15. Choose a second point in the feasible region that generates the same weekly profit as the first point.
- Draw a line through the two points.
  - Write the equation for this line in terms of  $x_1$  and  $x_2$ .

Every point on the line you have drawn generates the same weekly profit. For this reason, such a line is called a **line of constant profit**.

- Q16. Suppose Computer Flips generates \$6,000 of profit each week.
- Write an equation to represent this situation.
  - Graph this equation.

Note that the points (0, 20), (15, 10), and (30, 0) are on the line you drew, and each coordinate pair generates a profit of \$6,000 when substituted into the objective function.

- Q17. For each of the following profits, write an equation and then graph that equation (on the same coordinate plane).
- \$0 profit
  - \$3600 profit
  - \$4800 profit
  - \$7200 profit
- Q18. What do you notice about the three lines you have drawn?
- Q19. Which of the lines generates the largest weekly profit?
- Q20. If you were to continue drawing lines in this way, where does the line that generates the largest weekly profit intersect the feasible region? What is the profit at that point?

### 2.2.4 Solving the Problem

Hopefully, the previous line of investigation has suggested that the point or points representing the largest possible weekly profit are close to the boundary of the feasible region. That is, in order to maximize profits, Computer Flips' production rates should be as large as possible, while still keeping within the available installation time.

- Q21. Choose a point on the boundary of the feasible region, but not at a corner (vertex), and evaluate the profit there.
- Q22. Continue to choose points on the boundary, but try to increase the amount of profit each time.
- Q23. Finally, evaluate the profit at each of the corner points of the feasible region.
- Q24. What is the relationship between the corner points and the feasible region?
- Q25. What combination of products generates the greatest profit?

Notice that as the amount of constant profit increases, the lines are higher and further right in the first quadrant. Try to visualize a single line moving upward or to the right while its slope remains constant. The last point(s) in the feasible region that such a moving line touches will be optimal, because the profit is the greatest of any feasible points.

- Q26. There is not always only one optimal solution.  
Draw an example of a feasible region that could have more than one optimal solution.

### 2.2.5 Complicating the Problem

After several weeks of operation, one of the students in the sales department of Computer Flips does some market research. Based on this research, she decides that the company cannot sell more than 20 Simplex computers in any given week.

- Q27. Write an inequality that expresses this market constraint.
- Q28. Graph the new system of constraint inequalities.
- Q29. What do you notice about the optimal solution you found earlier?
- Q30. What is the optimal solution after adding the market constraint?

Now the students in the sales department of Computer Flips decide to extend the market research to the Omniplex model. On the basis of their research, they decide that Computer Flips cannot sell more than 16 Omniplex computers in any given week.

- Q31. Write an inequality for this new market constraint.

Graph the new feasible region.

Q32. Does the previous optimal solution lie in the new feasible region?

Q33. What is the optimal solution after the addition of the second market constraint?

The students at Computer Flips notice that they are getting a lot of returns. Every computer that was returned had a problem with one of the add-ons. They realize that they need to test their finished products before shipping them. They decide to assign the task of testing the computers to only one of the student installers. To accommodate this change, the other four student installers agree to work 10 hours per week, so that the total available installation time remains 40 hours per week. The student who will do the testing also works 10 hours per week. It takes her 20 minutes to test a Simplex and 24 minutes to test an Omniplex.

Q34. Write an inequality for the testing constraint based on the information in the previous paragraph.

Q35. Graph the new feasible region.

Q36. Using the new feasible region, what is the optimal solution?

Q37. Why is it possible to have a non-integer solution?

## 2.2.6 Success Breeds—An Even More Complicated Problem

Computer Flips has some initial success, so the students are considering producing two additional models: Multiplex and Megaplex. Multiplex will have more add-ons than Simplex, but not as many as Omniplex. Each Multiplex will generate \$250 profit. Megaplex, as the name implies, will have more add-ons than any of the other models. Each Megaplex will generate \$400 profit.

Q38. What are the decision variables in the new problem? What do they represent?

Q39. Write an equation for the profit ( $z$ ) the company would earn in a week.

The installation and testing times for each computer appear in Table 2.1.1. In addition, market research indicates that the *combined* sales of Simplex and Multiplex cannot exceed 20 computers per week, and the *combined* sales of Omniplex and Megaplex cannot exceed 16 computers per week.

	Simplex	Omniplex	Multiplex	Megaplex
Installation Time	60 min.	120 min.	90 min.	150 min.
Testing Time	20 min.	24 min.	24 min.	30 min.

**Table 2.2.1:** Installation and Testing times for all four computer models

Q40. Using the information above, formulate the constraints after the Multiplex and Megaplex models have been added to the product mix.

Q41. Is it possible to solve this problem by graphing? Why or why not?

In the next section, you will see another way to solve linear programming problems. In particular, the following section explores solving problems without graphing. You may wonder why this graphing approach cannot be used to solve every linear programming problem. If a problem contain three decision variables, it would be difficult for many people to visualize the graph. If a problem contains four or more decision variables, a graph is not even possible.

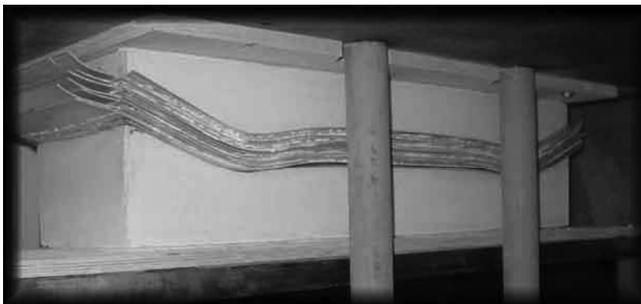
## Section 2.3: SK8MAN, Inc.

SK8MAN, Inc. manufactures and sells skateboards. A skateboard is made of a deck, two trucks that hold the wheels (see Figure 2.3.1), four wheels, and a piece of grip tape. SK8MAN, Inc. manufactures the decks of skateboards in its own factory and purchases the rest of the components.



**Figure 2.3.1:** A skateboard truck

To produce a skateboard deck, the wood must be glued and pressed, then shaped. After a deck has been produced, the trucks and wheels are added to the deck to complete a skateboard. Skateboard decks are made of either North American maple or Chinese maple. A large piece of maple wood is peeled into very thin layers called veneers. A total of seven veneers are glued at a gluing machine and then placed in a hydraulic press for a period of time (see Figure 2.3.2). After the glued veneers are removed from the press, eight holes are drilled for the truck mounts. Then the new deck goes into a series of shaping, sanding, and painting processes. Figure 2.3.3 shows a deck during the shaping process.



**Figure 2.3.2:** Maple veneers in a hydraulic press



**Figure 2.3.3:** Shaping a deck

Currently, SK8MAN, Inc. manufactures two types of skateboards: Sporty (Figure 2.3.4, top) and Fancy (Figure 2.3.4, bottom).



**Figure 2.3.4:** Sporty and Fancy skateboards

G. F. Hurley, the production manager at SK8MAN, Inc., needs to decide the production rate for each type of skateboard in order to make the most profit. Each Sporty board earns \$15 profit, and each Fancy board earns \$35 profit. However, Mr. Hurley might not be able to produce as many boards of either style as he would like, because some of the necessary resources, such as the North American maple and Chinese maple, are limited. That is, the production rates are *constrained* by the availability of the resources.

The Sporty board is a less expensive product, because its quality is not as good as the Fancy board. Chinese maple is used in the manufacture of Sporty decks. North American maple is used for Fancy decks. Because Chinese maple is soft, it is easier to shape. On average, it takes a worker 5 minutes to shape a Sporty board. However, a Fancy board requires 15 minutes to shape. G. F. Hurley needs to determine the production rates of Sporty and Fancy boards that will yield the maximum profit.

Q1. Develop a table to organize the information about Sporty and Fancy boards.

### 2.3.1 Problem Formulation

To find how to maximize profit, G. F. Hurley uses linear programming. The first step in the formulation of a linear programming problem is to define the decision variables in the problem.

Let:

$x_1$  represent the weekly production rate of Sporty boards and  
 $x_2$  represent the weekly production rate of Fancy boards.

The decision variables are then used to define the objective function. This function captures the goal in the problem, which, in this case, is to *maximize* the company's profits per week. Therefore, the objective function should represent the weekly profit from the sale of the two different styles of skateboards. The variable  $z$  is used to represent the amount of profit SK8MAN, Inc. earns per week.

Now, since the profit for each style of skateboard is known (\$15 and \$35, respectively), G. F. Hurley writes the objective function by expressing the profit ( $z$ ) in terms of the decision variables ( $x_1$  and  $x_2$ ):

$$\text{Maximize: } z = 15x_1 + 35x_2.$$

The last step in the formulation of the problem is to represent any constraints in terms of the decision variables. G. F. Hurley cannot just decide to make as many boards as he wants, because the number made is *constrained* by the available shaping time. Therefore, shaping time will be a constraint.

Suppose SK8MAN, Inc. is open for 8 hours a day, 5 days a week, which is a 40-hour workweek. However, since the information about shaping time is expressed in minutes, 40 hours is converted to 2,400 minutes. If SK8MAN, Inc. makes  $x_1$  Sporty boards and  $x_2$  Fancy boards per week, they use  $5x_1 + 15x_2$  minutes of shaping time.

For example, making 100 Sporty boards and 150 Fancy boards would take  $5(100) + 15(150) = 2,750$  minutes. Note that since 2,750 minutes is greater than 2,400 minutes, this production mix is not feasible.

Thus, the shaping time constraint is:

$$5x_1 + 15x_2 \leq 2400$$

There are also two not-so-obvious but completely logical constraints. G. F. Hurley knows the production rate cannot be a negative number for either type of skateboard, so he writes the non-negativity constraints:  $x_1 \geq 0$  and  $x_2 \geq 0$ .

The complete linear programming formulation looks like this:

### Decision Variables

Let:  $x_1$  = the weekly production rate of Sporty boards  
 $x_2$  = the weekly production rate of Fancy boards

### Objective Function

Maximize:  $z = 15x_1 + 35x_2$ ,  
 where  $z$  = the amount of profit SK8MAN, Inc. earns per week.

### Constraints

Subject to:  
 Shaping Time (min):  $5x_1 + 15x_2 \leq 2400$   
 Non-Negativity:  $x_1 \geq 0$  and  $x_2 \geq 0$

This formulated linear programming problem can now be solved graphically. To do so, G. F. Hurley sets up a coordinate plane with  $x_1$  as the horizontal axis and  $x_2$  as the vertical axis. Then, he graphs the constraints, as shown in Figure 2.3.5.

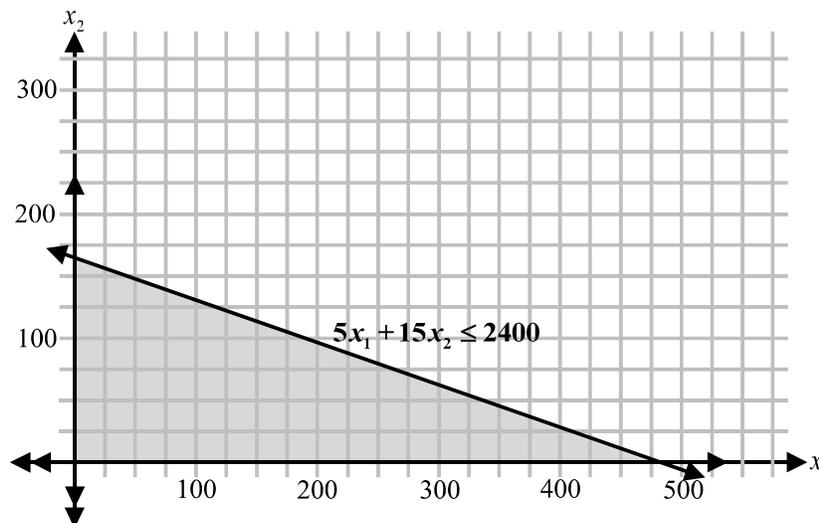
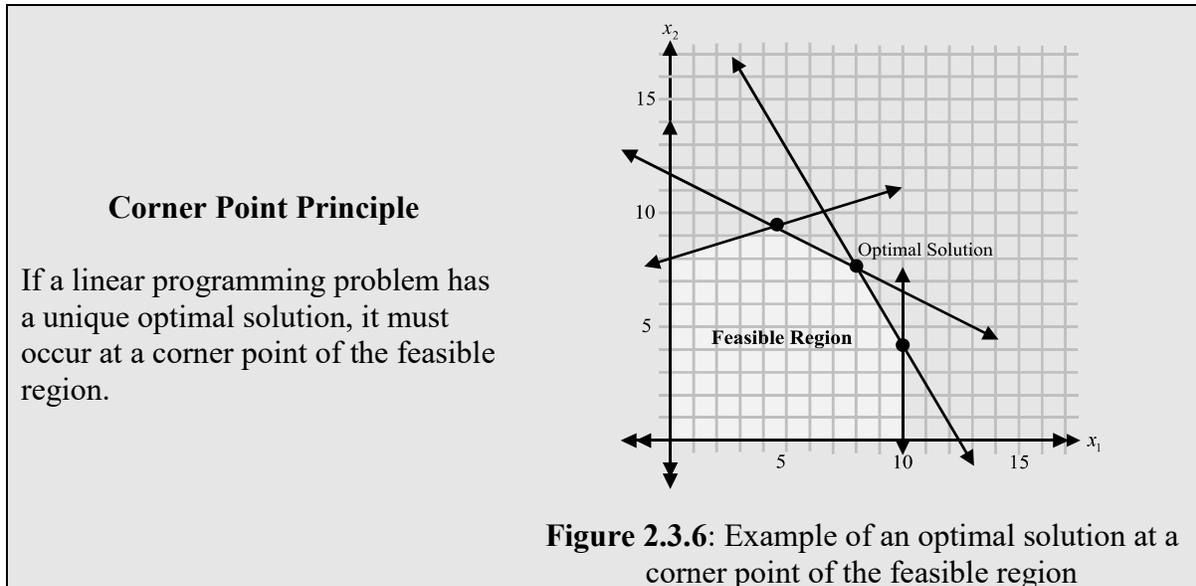


Figure 2.3.5: Graph of the system of constraints

G. F. Hurley recalls that every point in the shaded region satisfies all three constraints and is thus called the feasible region. Each ordered pair in the feasible region represents a combination of Fancy and Sporty boards that SK8MAN, Inc. could produce without violating any of the constraints. There are an infinite number of points in the feasible region, and the solution to the problem of maximizing profit is the one point that generates the most profit.

Rather than try to test an infinite number of points in the objective function, the optimal solution can be found by testing only a few points. This is due to the Corner Point Principle.



The Corner Point Principle allows us to simply evaluate the objective function at each corner point of the feasible region. Instead of there being an infinite number of possibilities for the optimal solution, there are only as many possibilities as there are corners of the feasible region.

Therefore, G.F. Hurley tests only the corner points of the feasible region in the objective function, as shown in Table 2.3.1.

Point	Profit
(0, 0)	$\$15(0) + \$35(0) = \$0$
(480, 0)	$\$15(480) + \$35(0) = \$7,200$
(0, 160)	$\$15(0) + \$35(160) = \$5,600$

**Table 2.3.1:** Corner points and their profits

Based on this information, SK8MAN, Inc. should produce 480 Sporty boards and 0 Fancy boards each week. This product mix will generate a weekly profit of \$7,200.

### 2.3.2 Add a New Constraint

G. F. Hurley just found out that the company that supplies the trucks for SK8MAN Inc.'s boards can provide at most 2,800 trucks per month. To make the problem easier, G. F. Hurley considers

a month to be four weeks, and therefore there are 700 trucks available per week. Since each skateboard needs two trucks, this new information represents another constraint.

The new complete linear programming formulation is as follows:

### Decision Variables

Let:  $x_1$  = the weekly production rate of Sporty boards  
 $x_2$  = the weekly production rate of Fancy boards

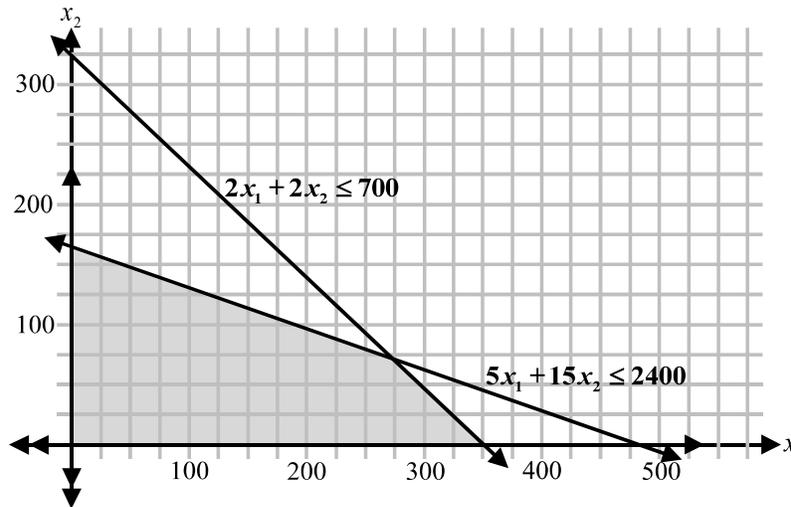
### Objective Function

Maximize:  $z = 15x_1 + 35x_2$ ,  
 where  $z$  = the amount of profit SK8MAN, Inc. earns per week.

### Constraints

Subject to:  
 Shaping Time (min):  $5x_1 + 15x_2 \leq 2400$   
 Trucks (#):  $2x_1 + 2x_2 \leq 700$   
 Non-Negativity:  $x_1 \geq 0$  and  $x_2 \geq 0$

Again, G. F. Hurley graphs the constraints, as shown in Figure 2.3.6.



**Figure 2.3.6:** The feasible region after adding the truck constraint

The new constraint changes the feasible region. The previous optimal solution is no longer included.

To find the new optimal solution, G. F. Hurley evaluates all the new corner points in the objective function, as seen in Table 2.3.2.

Point	Profit
(0, 0)	$\$15(0) + \$35(0) = \$0$
(350, 0)	$\$15(350) + \$35(0) = \$5,250$
(285, 65)	$\$15(285) + \$35(65) = \$6,550$
(0, 160)	$\$15(0) + \$35(160) = \$5,600$

**Table 2.3.2:** New corner points and their profits

Now G. F. Hurley can easily see that the maximum weekly profit SK8MAN, Inc. can earn is \$6,550, and the company does so by manufacturing 285 Sporty skateboards and 65 Fancy skateboards each week.

### 2.3.3 Add a Third Constraint

The U.S. Congress recently enacted legislation regulating the consumption of North American maple by U.S. manufacturers. As a consequence, SK8MAN, Inc.'s supplier told the company that it can provide no more than 840 veneers per week. The law leads to a new constraint. Recall that to make a skateboard, seven veneers are glued together and then placed in a hydraulic press (see Figure 2.3.2). Also recall that North American maple is used only for Fancy decks (Sporty decks are made from Chinese maple).

G. F. Hurley develops the new complete linear programming formulation:

#### Decision Variables

Let:  $x_1$  = the weekly production rate of Sporty boards  
 $x_2$  = the weekly production rate of Fancy boards

#### Objective Function

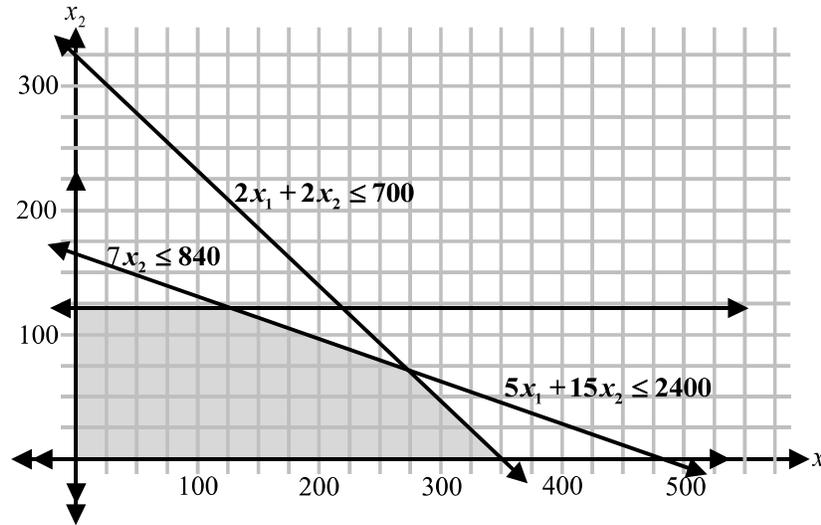
Maximize:  $z = 15x_1 + 35x_2$ ,  
 where  $z$  = the amount of profit SK8MAN, Inc. earns per week.

#### Constraints

Subject to:

Shaping Time (min):	$5x_1 + 15x_2 \leq 2400$
Trucks (#):	$2x_1 + 2x_2 \leq 700$
North American Maple (#):	$7x_2 \leq 840$
Non-Negativity:	$x_1 \geq 0$ and $x_2 \geq 0$

Again, G. F. Hurley graphs the constraints, as shown in Figure 2.3.7.



**Figure 2.3.7:** The feasible region after adding the North American maple constraint

When the new constraint is graphed, the feasible region changes, but the previous optimal solution (285 Sporty boards and 65 Fancy boards) is still included. Applying the corner point principle confirms that the maximum profit is unchanged because the optimal solution without the North American maple constraint remains in the feasible region after the North American maple constraint is added to the formulation. Table 2.3.3 shows the corner point calculations with the new constraint added to the formulation.

Point	Profit
(0, 0)	$\$15(0) + \$35(0) = \$0$
(350, 0)	$\$15(350) + \$35(0) = \$5,250$
(285, 65)	$\$15(285) + \$35(65) = \$6,550$
(120, 120)	$\$15(120) + \$35(120) = \$6,000$
(0, 120)	$\$15(0) + \$35(160) = \$5,600$

**Table 2.3.3:** Evaluating the objective function at each corner point of the new feasible region

Therefore, the optimal solution remains at 285 Sport boards and 65 Fancy boards. Since the North American maple constraint has no effect on the optimal product mix, it is called a **non-binding** constraint. The optimal product mix uses only  $65(7) = 455$  of the 840 available North American maple veneers (because the Sporty boards do not use North American maple, and the Fancy boards use 7 North American maple veneers per board). Not all of the available resource is expended in producing the optimal solution; thus there is a **slack** of  $840 - 455 = 385$ . The ideas of non-binding constraints and slack will be explored throughout the chapter.

### 2.3.4 Add a Fourth Constraint

Finally, SK8MAN, Inc.'s Chinese maple supplier has decided to limit its exports and will deliver a maximum of 1,470 veneers per week. Now G. F. Hurley needs to determine the new mix of products that will maximize weekly profit. As before, this information leads to a new constraint,

but the decision variables, objective function, and previous constraints remain the same. G. F. Hurley develops the new complete linear programming formulation:

### Decision Variables

Let:  $x_1$  = the weekly production rate of Sporty boards  
 $x_2$  = the weekly production rate of Fancy boards

### Objective Function

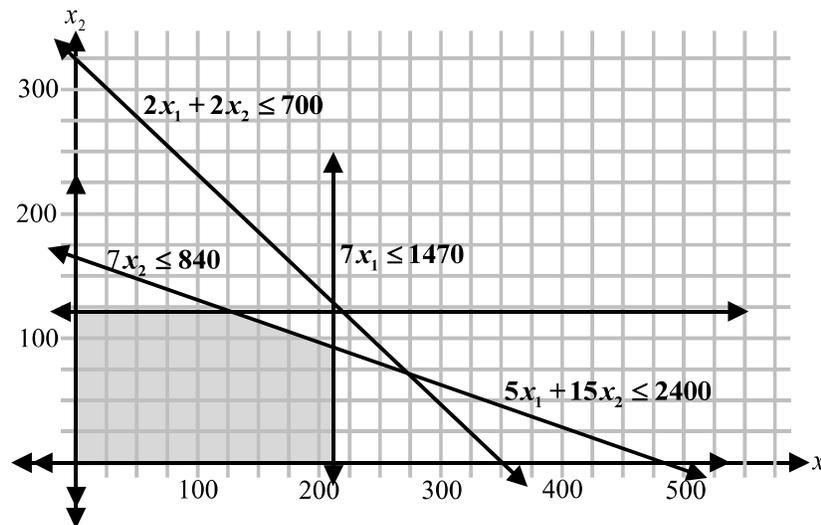
Maximize:  $z = 15x_1 + 35x_2$ ,  
 where  $z$  = the amount of profit SK8MAN, Inc. earns per week.

### Constraints

Subject to:

Shaping Time (min):  $5x_1 + 15x_2 \leq 2400$   
 Trucks (#):  $2x_1 + 2x_2 \leq 700$   
 North American Maple (#):  $7x_2 \leq 840$   
 Chinese Maple (#):  $7x_1 \leq 1470$   
 Non-Negativity:  $x_1 \geq 0$  and  $x_2 \geq 0$

Figure 2.3.8 shows the new graph of the constraints.



**Figure 2.3.8:** The feasible region after adding the Chinese maple constraint

Point	Profit
(0, 0)	$\$15(0) + \$35(0) = \$0$
(210, 0)	$\$15(210) + \$35(0) = \$3,150$
(210, 90)	$\$15(210) + \$35(90) = \$6,300$
(120, 120)	$\$15(120) + \$35(120) = \$6,000$
(0, 120)	$\$15(0) + \$35(120) = \$5,600$

**Table 2.3.4:** Evaluating the objective function after the last constraint is added

As the graph in Figure 2.3.8 shows, the feasible region changes again. The previous optimal solution, (285, 65), is no longer feasible, so each corner point must be tested. Table 2.3.4 presents the profit associated with each corner point feasible solution. Now SK8MAN, Inc.'s maximum profit is \$6,300 per week. The product mix that achieves that profit is 210 Sporty skateboards and 90 Fancy skateboards. Notice the tendency for maximum profit to decrease as the number of constraints increases.

### 2.3.5 Add a Third Decision Variable

SK8MAN Inc. is introducing a new product—the Pool-Runner skateboard—which is made from Chinese maple. It is wider and shorter than the Sporty board so that it will be easy to use in a pool. It takes four minutes to shape a Pool-Runner board, and SK8MAN, Inc. earns \$20 for each one sold. G. F. Hurley needs to determine the new constraints and the optimal product mix. He begins by developing the new complete linear programming formulation:

- Q2. Develop a table to organize the information about Sporty, Fancy, and Pool-Runner boards.

#### Decision Variables

Let:  $x_1$  = the weekly production rate of Sporty boards  
 $x_2$  = the weekly production rate of Fancy boards  
 $x_3$  = the weekly production rate of Pool-Runner boards

#### Objective Function

Maximize:  $z = 15x_1 + 35x_2 + 20x_3$ , where  $z$  = SK8MAN, Inc.'s weekly profit.

#### Constraints

Subject to:

Shaping Time (min):	$5x_1 + 15x_2 + 4x_3 \leq 2400$
Trucks (#):	$2x_1 + 2x_2 + 2x_3 \leq 700$
North American Maple (#):	$7x_2 \leq 840$
Chinese Maple (#):	$7x_1 + 7x_3 \leq 1470$
Non-Negativity:	$x_1 \geq 0, x_2 \geq 0, \text{ and } x_3 \geq 0$

Adding the third decision variable makes solving this problem graphically very difficult. Graphing the feasible region with three decision variables would require three dimensions. While it is possible to do so, visualizing such a feasible region is very difficult for most people. There are two other possible ways to solve linear programming problems involving three or more decision variables. The first way is to apply a paper-and-pencil technique called the Simplex Method. This method will not be described here. Instead, the use of a spreadsheet Solver will be explored. A spreadsheet Solver applies a computer procedure to solve linear programming problems. The following directions will walk you through the steps needed to use the Solver function in Microsoft Excel to solve this problem.

### 2.3.6 Excel Solver and SK8MAN

Here we describe how to use Solver, an add-in to Excel, to solve linear programming problems. The steps involved are outlined below. It begins with making sure that Solver is an active add-in to Excel. There are then two major phases. The first phase comprises the steps for setting up the spreadsheet to represent all of the information in the linear programming formulation. The second phase encompasses the steps for setting up Solver to execute an algorithm to optimize the linear programming problem.

0. Install Excel Add-in “Solver”

#### Phase I. Set up spreadsheet

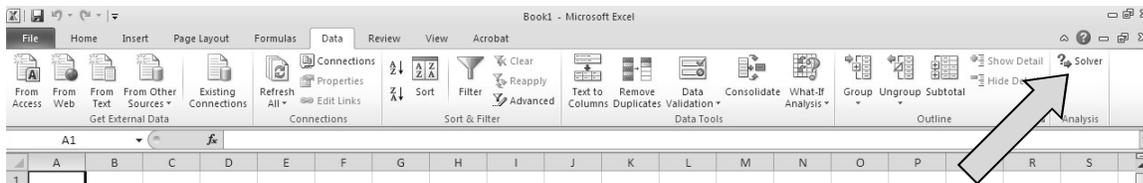
1. Input the components of the linear programming formulation
2. Write the equation for the objective function
3. Write the equations for the constraints

#### Phase II. Set up Solver

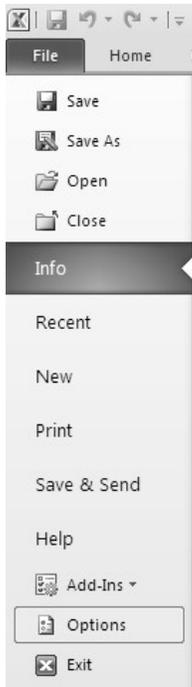
4. Open Solver window and specify location of objective function
5. Specify problem is maximization
6. Specify location of values of decision variables
7. Add in the regular constraints
8. Specify all decision variables are non-negative
9. Specify solution method
10. Select the appropriate options
11. Solve and Print Reports

#### Step 0: Add-in Solver

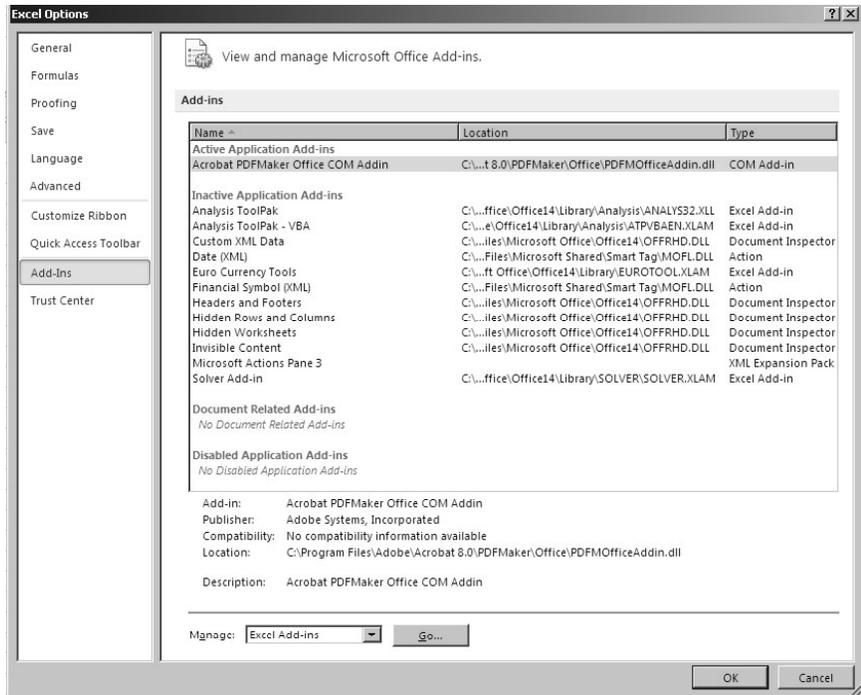
Open Excel and click on the “Data” tab. The Solver option should be at the top right of the menu (see Figure 2.3.9). If it is not available, it needs to be added. To add Solver, go to “Options” under the “File” menu (see Figure 2.3.10) and click on “Add-Ins” (see Figure 2.3.11). Next, choose the “Solver Add-in” and click “Go.” Finally, check “Solver Add-in” and click “OK” (See Figure 2.3.12). It may take some time to install.



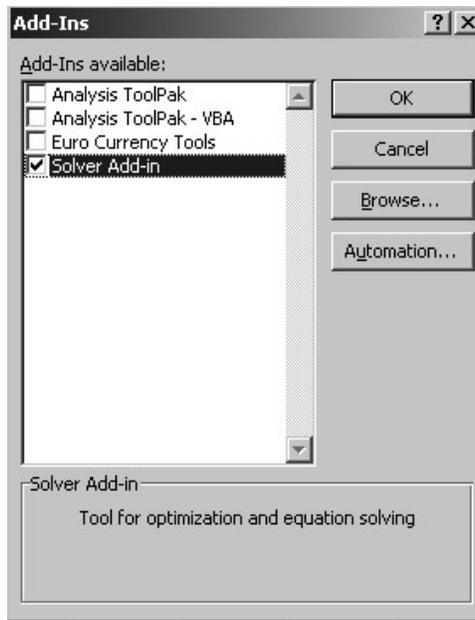
**Figure 2.3.9:** Location of Solver in Microsoft Excel 2010



**Figure 2.3.10:** Choose “Options” under the “File” menu



**Figure 2.3.11:** The Add-Ins menu in Microsoft Excel 2010



**Figure 2.3.12:** Choose “Solver Add-in” and click “OK”

	A	B	C	D	E	F	G
1	Chapter 2 LP Maximization						
2	2.2 SK8MAN Inc.						
3	Profit Maximization Problem						
4		Sporty	Fancy	Pool-Runner			
5	Decision Variables	X1	X2	X3			
6	Decision Values [weekly production rate]						
7					Total Profit		
8	Objective Function [Profit (\$)]	15	35	20			
9							
10	Constraints				Amount Used		Amount Available
11	Shaping Time (minutes)	5	15	4		<=	2400
12	Truck Availability	2	2	2		<=	700
13	North American Maple Veneers	0	7	0		<=	840
14	Chinese Maple Veneers	7	0	7		<=	1470

**Figure 2.3.13:** Setting up the problem formulation in an Excel spreadsheet

### Step 1: Set Up a Spreadsheet

To set up the spreadsheet, keep in mind the complete linear programming formulation of the SK8MAN problem with three decision variables. Begin by putting a title in column A (see Figure 2.3.13). This description will make things easier when coming back to this spreadsheet at some later date.

In planning the input of the elements of the linear programming model, it is important to understand that there are 3 components of each equation: coefficients, decision variables and equation. For example, the objective function for the three decision Snowboard problem is

$$z = 15x_1 + 35x_2 + 20x_3,$$

This equation has 3 elements.

1. A coefficient for each decision variable that represents its profit margin
2. The individual values of the decision variable represented by  $x_i$
3. The mathematical equation which is the sum total of the product of each coefficient and each decision variable.

In Excel, the coefficients for the objective function are placed in a separate row from the values of the decision variables. Then a formula is written into a cell to perform the calculation. The same three elements apply to each constraint; the coefficients represent the rate at which a resource is consumed by one unit of that decision variable.

In the setup presented in Figure 2.3.13, Row 4, columns B, C, D contain the original names of the decision variables. Row 6 lists the corresponding representations,  $X_1$ ,  $X_2$ , and  $X_3$ , used in the linear programming formulation. The information in Rows 4 and 5 is simply for clarification. Row 6 will contain the actual values of the decision variables that Solver will optimize. The cells are initially empty and treated as zeroes. Because the values in these cells are continually

changing, Solver calls them “changing cells” or “changing variable cells.” To identify the rows of the spreadsheet, we type in “Decision Variables” in A5 and “Decision Values” in A6.

Row 8 columns B, C, and D contain the coefficients of the objective function, the profit margin for each product. The term “Objective Function” is written in cell A8. Cell E8 will contain the formula for the total profit with a label “Total Profit” placed in the cell above, E7.

Next, we use column A to label each of the for constraints that are to be recorded in rows 11 through 14.

Now, the coefficients for each of the constraints are entered in rows 11 through 14, columns B through D. The direction of each constraint inequality, all  $\leq$  in this case, is entered in cells F11-F14. This reminds us of the nature of each individual constraint but is not actually used in any computations. Finally, the resources available that appear on right-hand side of each constraint are entered in cells G11-G14. We insert “Constraints” and “Amount Available” in cells A10 and G10 to identify the information that appears below. This is all displayed in Figure 2.3.13. Lastly, we write “Amount Used” in cell E10, to let us know that cells E11-E14 will contain the formula that calculate the amount of each resource used by the set of decision variables. This is discussed in step 3.

### **Step 2: Develop a Formula for the Objective Function**

The objective function must be defined mathematically, so that the Solver software can compute its value. This is accomplished by entering a formula into cell E8. Notice that the label “Total Profit” is directly above this cell.

First, recall that the objective function is  $z = 15x_1 + 35x_2 + 20x_3$ . To compute its value, 15 is multiplied by the weekly production rate of Sport skate boards, 35 is multiplied by the weekly production rate of Fancy skateboards, and 20 is multiplied by the weekly production rate of Pool-Runner skateboards. The objective function coefficients are stored in row 8 and the values of the decision variables in row 6. The total profits for each product is computed and then the profits are summed by typing the formula “=B6\*B8+C6\*C8+D6\*D8 in cell E8 (see Figure 2.3.14, and notice that the formula is displayed next to  $f_x$  in the bar above the horizontal ruler).

E8		=B6*B8+C6*C8+D6*D8					
	A	B	C	D	E	F	G
1	Chapter 2 LP Maximization						
2	2.2 SK8MAN Inc.						
3	Profit Maximization Problem						
4		Sporty	Fancy	Pool-Runner			
5	Decision Variables	X1	X2	X3			
6	Decision Values [weekly production rate]						
7					Total Profit		
8	Objective Function [Profit (\$)]	15	35	20	\$0.00		
9							
10	Constraints				Amount Used		Amount Available
11	Shaping Time (minutes)	5	15	4		<=	2400
12	Truck Availability	2	2	2		<=	700
13	North American Maple Veneers	0	7	0		<=	840
14	Chinese Maple Veneers	7	0	7		<=	1470

Figure 2.3.14: The formula for the objective function

### Step 3: Develop Formulas for the Left-Hand Side of Each Constraint

The formulas for the left-hand sides of the constraints are written the same way as the formula for the objective function. Again, the coefficients are multiplied by the cells containing the weekly production rate of each skateboard, namely B6, C6, and D6, and then added together. These formulas are typed in cells E11-E14. In Figure 2.3.15, we have highlighted cell E11 which contains the formula for the shaping constraint “=B6\*B11+C6\*C11+D6\*D11. Notice B6, C6, and D6 appear again since they correspond to the values of the decision variables. (see Figure 2.3.15). Note that these expressions compute the sum total of each resource that is consumed by a particular set of values of the decision variables. Currently, cells E11-E14 report a value of 0 because all of the decision variables are still 0.

E11		=B6*B11+C6*C11+D6*D11					
	A	B	C	D	E	F	G
1	Chapter 2 LP Maximization						
2	2.2 SK8MAN Inc.						
3	Profit Maximization Problem						
4		Sporty	Fancy	Pool-Runner			
5	Decision Variables	X1	X2	X3			
6	Decision Values [weekly production rate]						
7					Total Profit		
8	Objective Function [Profit (\$)]	15	35	20	\$0.00		
9							
10	Constraints				Amount Used		Amount Available
11	Shaping Time (minutes)	5	15	4	0.0	<=	2400
12	Truck Availability	2	2	2	0.0	<=	700
13	North American Maple Veneers	0	7	0	0.0	<=	840
14	Chinese Maple Veneers	7	0	7	0.0	<=	1470

Figure 2.3.15: Formulas for the constraint left-hand sides are entered

At this point, G. F. Hurley decided to explore the spreadsheet and the formulas in the spreadsheet. He considered a production plan of 40 Sporty, 90 Fancy and 60 Pool-Runner

skateboards (See Figure 2.3.16). This solution is feasible because each of the resources used is less than the amount available. For example, 1790 minutes of shaping time is used which is less than the 2400 minutes that are available. The total profit for this solution is \$4,950.

Mr. Hurley then explored the impact of doubling production of the highly profitable Fancy skateboards to 180 per week (See Figure 2.3.17). The profit for this plan would be \$8,100. However, when he checked the resource constraints, he found this plan was infeasible. This plan required 3,140 minutes of shaping time which is more than the available shaping time minutes. It also needed 1,260 North American Maple Veneers but there were only 840 available each week.

	A	B	C	D	E	F	G
1	Chapter 2 LP Maximization						
2	2.2 SK8MAN Inc.						
3	Profit Maximization Problem						
4		Sporty	Fancy	Pool-Runner			
5	Decision Variables	X1	X2	X3			
6	<b>Decision Values [weekly production rate]</b>	40	90	60			
7					<b>Total Profit</b>		
8	<b>Objective Function [Profit (\$)]</b>	15	35	20	<b>\$4,950.00</b>		
9							
10	<b>Constraints</b>				<b>Amount Used</b>		<b>Amount Available</b>
11	Shaping Time (minutes)	5	15	4	<b>1,790.0</b>	<=	2400
12	Truck Availability	2	2	2	<b>380.0</b>	<=	700
13	North American Maple Veneers	0	7	0	<b>630.0</b>	<=	840
14	Chinese Maple Veneers	7	0	7	<b>700.0</b>	<=	1470

**Figure 2.3.16:** The spreadsheet showing a feasible solution

	A	B	C	D	E	F	G
1	Chapter 2 LP Maximization						
2	2.2 SK8MAN Inc.						
3	Profit Maximization Problem						
4		Sporty	Fancy	Pool-Runner			
5	Decision Variables	X1	X2	X3			
6	<b>Decision Values [weekly production rate]</b>	40	180	60			
7					<b>Total Profit</b>		
8	<b>Objective Function [Profit (\$)]</b>	15	35	20	<b>\$8,100.00</b>		
9							
10	<b>Constraints</b>				<b>Amount Used</b>		<b>Amount Available</b>
11	Shaping Time (minutes)	5	15	4	<b>3,140.0</b>	<=	2400
12	Truck Availability	2	2	2	<b>560.0</b>	<=	700
13	North American Maple Veneers	0	7	0	<b>1,260.0</b>	<=	840
14	Chinese Maple Veneers	7	0	7	<b>700.0</b>	<=	1470

**Figure 2.3.17:** The spreadsheet showing an infeasible solution

#### Step 4: Open Solver

Before the problem can be solved, parameters need to be set up in the Solver program. These parameters include all of the parts of the problem formulation: the decision variables, the objective function, and the constraints. Solver needs to be told where in the spreadsheet each of these parameters is located. First, click on the cell containing the objective function (cell E8). Second, go to the Data menu and choose Solver (see Figure 2.3.18). A Solver Parameters window should come up showing \$E\$8 as the target cell (see Figure 2.3.19). Notice that the

target cell is the cell in which the objective function is defined. The dollar signs merely indicate that specific cell. When the Solver Parameters window is opened, if cell E8 is not already selected, you can type E8 with or without the \$ signs.

E18		fx =B6*B8+C6*C8+D6*D8					
	A	B	C	D	E	F	G
1	Chapter 2 LP Maximization						
2	2.2 SK8MAN Inc.						
3	Profit Maximization Problem						
4		Sporty	Fancy	Pool-Runner			
5	Decision Variables	X1	X2	X3			
6	Decision Values [weekly production rate]						
7					Total Profit		
8	Objective Function [Profit (\$)]	15	35	20	\$0.00		
9							
10	Constraints				Amount Used		Amount Available
11	Shaping Time (minutes)	5	15	4	0.0	<=	2400
12	Truck Availability	2	2	2	0.0	<=	700
13	North American Maple Veneers	0	7	0	0.0	<=	840
14	Chinese Maple Veneers	7	0	7	0.0	<=	1470
15							

Figure 2.3.18: Cell E8 highlighted when Solver is opened

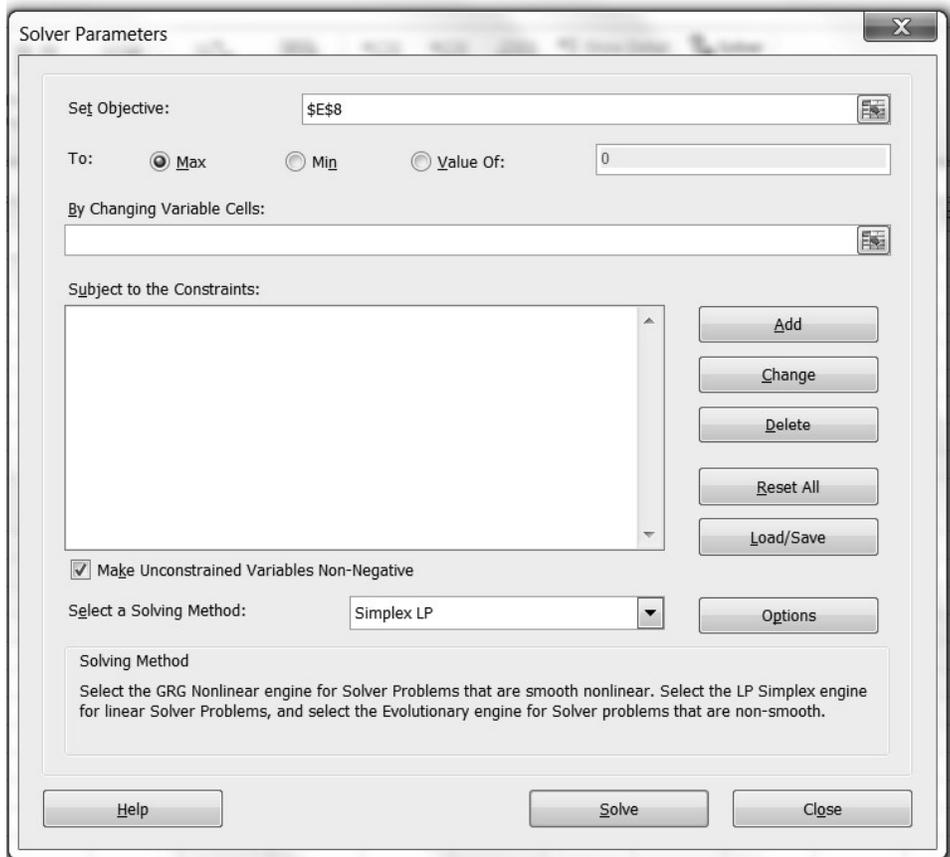


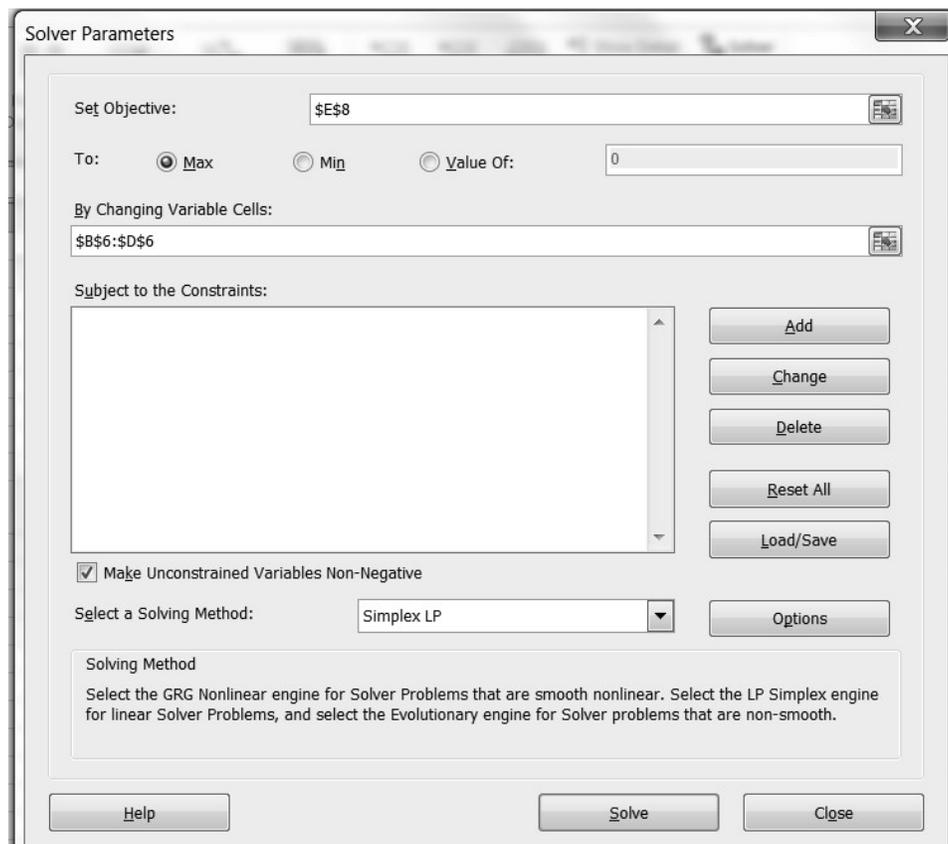
Figure 2.3.19: Solver Parameters dialogue window when Cell E8 is highlighted in the spreadsheet

### Step 5: Choose the Type of Linear Programming Problem

Recall that the objective for this problem is to *maximize* profit. Therefore, in the Solver Parameters window, make sure the “Max” circle is filled in (see Figure 2.3.20).

### Step 6: Identify the Decision Variable Cells – “By Changing Variable Cells”

Next, look at the “By Changing Variable Cells” title. Solver needs to be told that the decision variable values are in cells B6, C6, and D6. To do so, you could type in B6, C6, D6. However, it is preferable to use the shortcut B6:E6. In Excel a semicolon “;” between two cells, means use all cells in the range from B6 to E6. Instead of typing in this range, you can also click on cell B6 and drag the pointer across to E6. Solver will add dollar signs in the cell names, and you can leave them as they are (see Figure 2.3.20).



**Figure 2.3.20:** Identifying the decision variable cells – “By Changing Variable Cells”

### Step 7: Set Up the Constraints

Click inside the box labeled “Subject to the Constraints.” The constraints will be added, one at a time. Click “Add.” As shown in Figure 2.3.21, a window will appear titled “Add Constraint.”

For the shaping constraint in row 11, the value of the formula in cell E11 should be less than or equal to the value in cell G11. The amount of shaping time used needs to be less than or equal to 2400 minutes. To do this, type F11 (or click on F11) into “Cell Reference” and type G11 into “Constraint” (or click on G11). Use the drop-down menu to change the inequality symbol, if

necessary. The default is  $\leq$  which applies to each of the constraints in the SK8MAN example. Click “Add” and then continue to the next constraint. After adding the last constraint, click OK. Figure 2.3.22 shows the Solver Parameters window after all four constraints have been added.

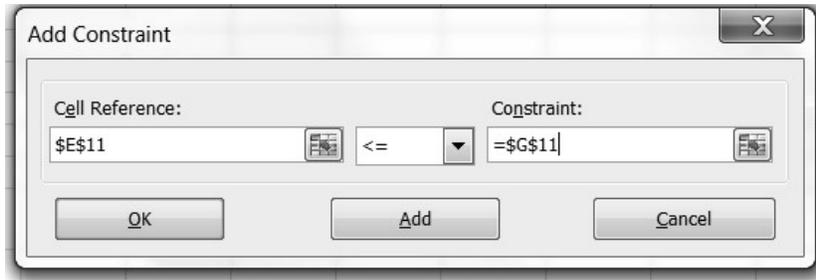


Figure 2.3.21: Drop-down menu for adding a constraint

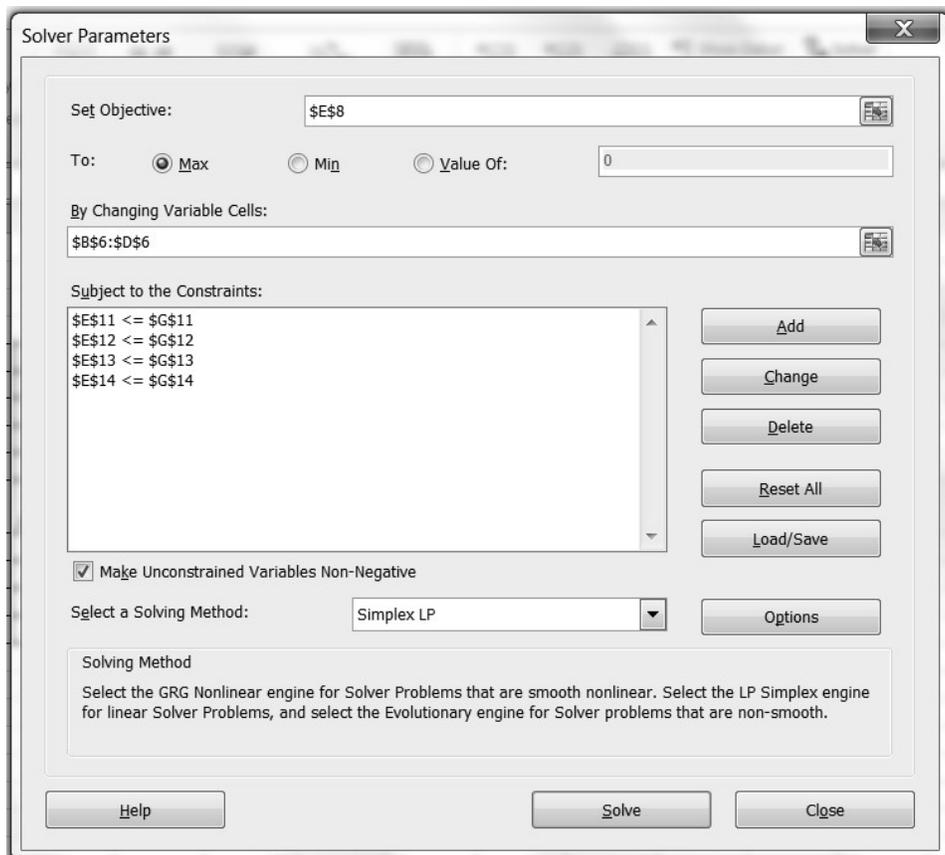


Figure 2.3.22: Solver parameters window with four constraints added

### Step 8: Verify Non-Negativity Constraints

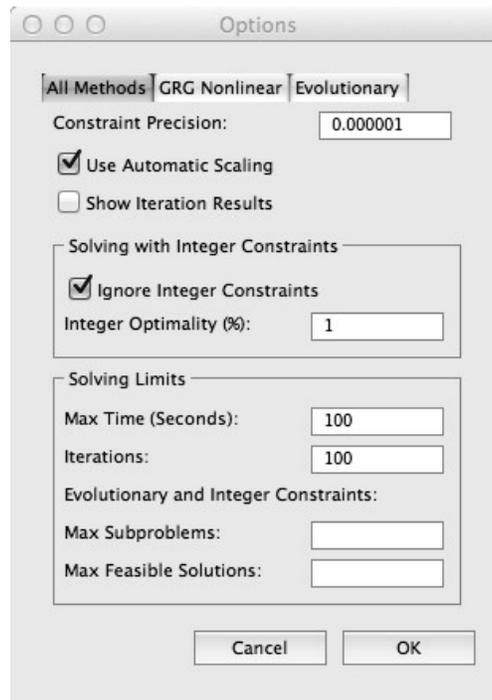
Notice that the non-negativity constraints were not included in the spreadsheet setup because Solver has a shortcut for doing so. Under the “Subject to the Constraints” box, make sure “Make Unconstrained Variables Non-Negative” is checked (see Figure 2.3.22).

**Step 9: Select Solving Method**

Next, a method of solution must be selected. If the box next to “Select a Solving Method” does *not* say “Simplex LP”, use the drop-down menu to select “Simplex LP” (See Figure 2.3.22).

**Step 10: Set Up the Solver Options**

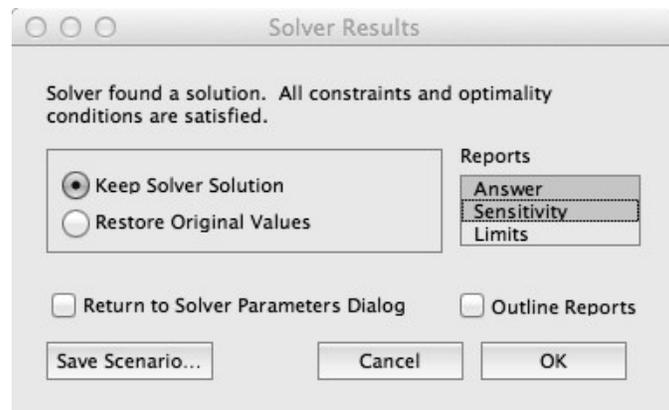
Click on the “Options” button in the Solver Parameters window (see Figure 2.3.22). In the “All Methods” tab, check the box that says “Use Automatic Scaling.” Also check the box that says “Ignore Integer Constraints.” Solution precision can be controlled to some extent using the “Constraint Precision” option. For most examples, the default setting (0.000001) will suffice. Finally, you can set “Solving Limits.” Setting “Max Time (Seconds)” to 100 and “Iterations” to 100 works well for most problems. Figure 2.3.23 shows these settings. Click “OK” to finish.



**Figure 2.3.23:** Adding the Solver Options

**Step 11: Solve and Print Reports**

Finally, click “Solve.” The Solver Results window appears, and the results can be seen in the spreadsheet. Answer and Sensitivity Reports are generated by clicking on the name of the report in the window, as shown in Figure 2.3.24. Solver then creates a new tab that corresponds to a worksheet for each report.



**Figure 2.3.24:** Generating Answer and Sensitivity Reports in Solver

To review, the steps for solving a maximization linear programming problem using Excel Solver are given in Table 2.3.5.

Step	Description
0	Add in Solver (skip this step once Solver has been added).
1	Set up a spreadsheet using the linear programming formulation.
2	Develop a formula for the objective function.
3	Develop formulas for left-hand sides of the constraints, label the type of constraint ( $\leq$ , $=$ , $\geq$ ), and type in the values for the right-hand sides of the constraints.
4	Open Solver from the Data menu.
5	Choose the type of linear programming problem.
6	Identify the decision variable cells.
7	Set up the constraints.
8	Verify non-negativity constraints.
9	Select solving method from the drop-down menu.
10	Set up the Solver options.
11	“Solve” and generate Answer and Sensitivity Reports.

**Table 2.3.5:** Steps for solving a linear programming problem using Excel Solver

You might also look at the YouTube videos we developed. There are two file in the playlist: - excel basics and -excel solver tutorial

<http://www.youtube.com/playlist?list=PLBcLt4kL3jChKdrE2ex5RV0OWymb70gAk>

### 2.3.7 Three Excel Shortcuts

There are three shortcuts that speed up the process of creating the formula and setting up Solver. The shortcuts also reduce the likelihood of making an error.

**Shortcut 1.** The copy and paste feature of the spreadsheet can be used to speed up the process of adding the constraint formulas. When all of those formulas have the same structure as the formula for the objective function, the objective function formula can simply be copied and pasted into each of the cells containing the left-hand side of a constraint. Before copying the formula, however, the row numbers for the decision variables must be locked, so that Solver will always refer to the specific row containing the values of the decision variables. In our example, row 6 is locked by inserting a \$ to the left of the 6 in each term of the objective function formula. When finished, the formula looks like this: =B8\*B\$6+C8\*C\$6+D8\*D\$6. Now, when this formula is pasted into cells E11-E14, the row number of the first factor in each term of the formula will be changed to the current row number. However, the row number of the second factor in each term is locked at row 6 where the decision variables are. For example, when pasted into row 11, the formula will read: =B11\*B\$6+C11\*C\$6+D11\*D\$6.

**Shortcut 2.** The second shortcut uses a built-in spreadsheet function, SUMPRODUCT. The SUMPRODUCT function allows you to multiply each value in one array by a corresponding value in an equal sized array and sum all of the products. To write the formula for the objective function simply type in cell F8.

=SUMPRODUCT(B6:D6,B8:D8)

The colon between B6 and D6 tells Excel to use every value in the range of cells between B6 and D6. After placing a comma, type in the second range B8:D8. Be sure to have parentheses before and after the ranges.

E8		=SUMPRODUCT(B\$6:D\$6,B8:D8)					
	A	B	C	D	E	F	G
1	Chapter 2 LP Maximization						
2	2.2 SK8MAN Inc.						
3	Profit Maximization Problem						
4		Sporty	Fancy	Pool-Runner			
5	Decision Variables	X1	X2	X3			
6	Decision Values [weekly production rate]						
7					Total Profit		
8	Objective Function [Profit (\$)]	15	35	20	\$0.00		
9							
10	Constraints				Amount Used		Amount Available
11	Shaping Time (minutes)	5	15	4	0.0	<=	2400
12	Truck Availability	2	2	2	0.0	<=	700
13	North American Maple Veneers	0	7	0	0.0	<=	840
14	Chinese Maple Veneers	7	0	7	0.0	<=	1470

Figure 2.3.25: SUMPRODUCT Function

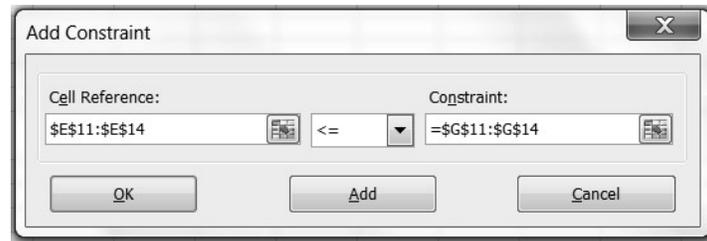
We are going to copy and paste cell E8 into cells E11-E14. We need to insert the \$ sign to establish row 6, the decision variable row, as unchanging.

=SUMPRODUCT(B\$6:D\$6,B8:D8)

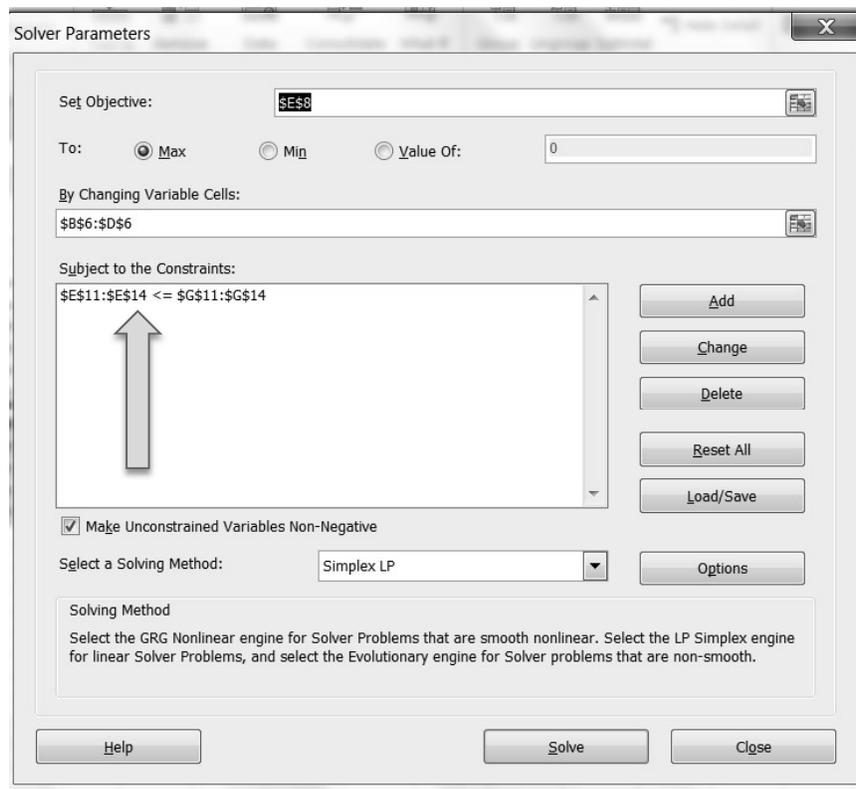
Now the formula in cell E8 can be copied and pasted into cells E11-E14 for the constraint left-hand side values. Cell F11 should now contain

=SUMPRODUCT(B\$6:D\$6,B11:D11)

**Shortcut 3.** In the Solver Parameters window, it is possible to add a group of constraints of the same type in one step. In this example all of the constraints are of the type  $\leq$ . To use this shortcut, after clicking “Add” in the Solver Parameters window, type in a range of cells in each box. In the “Cell Reference” box record D11:D14 and in the “Constraint” box record G11:G14 boxes (see Figure 2.3.26). Solver interprets these ranges as meaning E11 must be less than or equal to G11, E12 must be less than or equal to G12, etc. Figure 2.3.27 shows how this changes the way constraints are listed in the Solver Parameter window.



**Figure 2.3.26:** Entering four constraints at one time.



**Figure 2.3.27:** Solver Parameters with all four constraints at one time.

## Section 2.4: The Pallas Sport Shoe Company

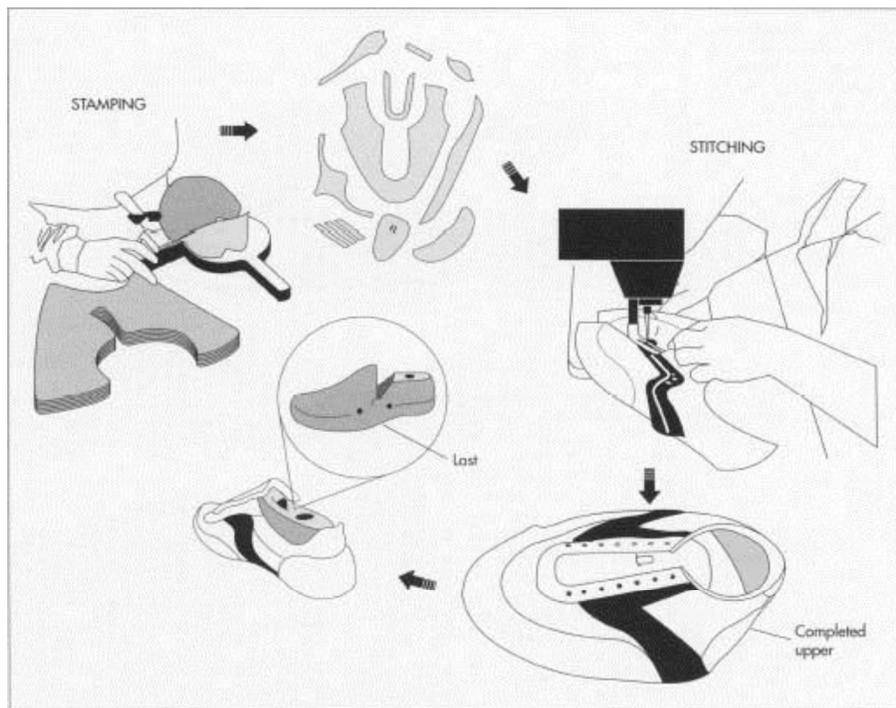
The Pallas Sport Shoe Company manufactures six different lines of sport shoes: High Rise, Max-Riser, Stuff It, Zoom, Sprint, and Rocket. Table 2.4.1 displays the amount of profit generated by each pair of shoes for each of these six lines. The production manager of the company would like to determine the daily production rates for each line of shoes that will maximize profit.

Product	High Rise	Max-Riser	Stuff It	Zoom	Sprint	Rocket
Profit	\$18	\$23	\$22	\$20	\$18	\$19

**Table 2.4.1:** Profit per pair for six lines of sport shoes

There are six main steps in the production of a pair of sport shoes at Pallas. Some of these steps can be seen in Figure 2.4.1.

1. **Stamping:** The parts that go together to form the upper portion of the shoe are cut using patterns on a large stamping machine. This process resembles cutting dough with a cookie cutter.
2. **Upper Finishing:** These parts are stitched or cemented together to form an upper, and holes for the laces are punched.
3. **Insole Stitching:** An insole is stitched to the sides of the upper.
4. **Molding:** The completed upper is then placed on a plastic mold, called a *last*, to form the final shape of the shoe.
5. **Sole-to-Upper Joining:** After the upper has been molded, it is cemented to the bottom sole using heat and pressure.
6. **Inspecting:** Finally, the shoe is inspected, and any excess cement is removed.



**Figure 2.4.1:** The steps in manufacturing a sport shoe

The time it takes to complete each of these steps differs across the six lines, and the total time available for each process constrains the daily production rates. Table 2.4.2 shows the time, in minutes, required for each of the six production steps for each of the six lines of sport shoe produced by Pallas as well as the total number of minutes available per day for each step.

	High Rise	Max-Riser	Stuff It	Zoom	Sprint	Rocket	Total Time Available
Stamping	1.25	2	1.5	1.75	1	1.25	420
Upper Finishing	3.5	3.75	5	3	4	4.25	1,260
Insole Stitching	2	3.25	2.75	2.25	3	2.5	840
Molding	5.5	6	7	6.5	8	5	2,100
Sole-to-Upper Joining	7.5	7.25	6	7	6.75	6.5	2,100
Inspecting	2	3	2	3	2	3	840

**Table 2.4.2:** Time, in minutes, per production step for each line of shoes and total time available

Q1. Based on the information in Tables 2.4.1 and 2.4.2, predict what the optimal solution will be for this problem. Explain your reasoning.

### 2.4.1 Problem Formulation

The linear programming formulation of the Pallas Sport Shoe Company problem appears below.

#### Decision Variables

Let:

- $x_1$  = the daily production rate of High Rise
- $x_2$  = the daily production rate of Max-Riser
- $x_3$  = the daily production rate of Stuff It
- $x_4$  = the daily production rate of Zoom
- $x_5$  = the daily production rate of Sprint
- $x_6$  = the daily production rate of Rocket

#### Objective Function

Maximize:  $z = 18x_1 + 23x_2 + 22x_3 + 20x_4 + 18x_5 + 19x_6$ ,  
 where  $z$  = the amount of profit Pallas Sport Shoe Company earns per day

#### Constraints

Subject to:

Stamping Time:  $1.25x_1 + 2x_2 + 1.5x_3 + 1.75x_4 + x_5 + 1.25x_6 \leq 420$   
 Upper Finishing Time:  $3.5x_1 + 3.75x_2 + 5x_3 + 3x_4 + 4x_5 + 4.25x_6 \leq 1,260$   
 Insole Stitching Time:  $2x_1 + 3.25x_2 + 2.75x_3 + 2.25x_4 + 3x_5 + 2.5x_6 \leq 840$   
 Molding Time:  $5.5x_1 + 6x_2 + 7x_3 + 6.5x_4 + 8x_5 + 5x_6 \leq 2,100$   
 Sole-to-Upper Joining Time:  $7.5x_1 + 7.25x_2 + 6x_3 + 7x_4 + 6.75x_5 + 6.5x_6 \leq 2,100$   
 Inspecting Time:  $2x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6 \leq 840$   
 Non-Negativity:  $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0$

Figure 2.4.2 contains this formulation in an Excel spreadsheet format for use with Solver.

	A	B	C	D	E	F	G	H	I	J
1	Chapter 2: LP Maximization									
2	2.3 Pallas Sport Show Company									
3	Profit Maximization									
4										
5	Decision Variable	<b>High Rise (<math>x_1</math>)</b>	<b>Max-Riser (<math>x_2</math>)</b>	<b>Stuff It (<math>x_3</math>)</b>	<b>Zoom (<math>x_4</math>)</b>	<b>Sprint (<math>x_5</math>)</b>	<b>Rocket (<math>x_6</math>)</b>			
6	Decision Values [daily production rate]									
7										<b>Total Profit</b>
8	Objective Function [Profit (\$)]	18	23	22	20	18	19			\$0.00
9										
10	<b>Constraints</b>							<b>Used</b>		<b>Available</b>
11	Cutting Time (minutes)	1.25	2	1.5	1.75	1	1.25	0	≤	420
12	Upper Finishing Time (minutes)	3.5	3.75	5	3	4	4.25	0	≤	1260
13	Insole Stitching Time (minutes)	2	3.25	2.75	2.25	3	2.5	0	≤	840
14	Molding Time (minutes)	5.5	6	7	6.5	8	5	0	≤	2100
15	Sole-to-Upper Joining Time (minutes)	7.5	7.25	6	7	6.75	6.5	0	≤	2100
16	Inspecting Time (minutes)	2	3	2	3	2	3	0	≤	840

Figure 2.4.2: An Excel spreadsheet formulation of the Pallas Shoe problem

## 2.4.2 Problem Solution

After solving this linear programming problem in Excel, an Answer Report can be generated, as shown in Figure 2.4.3. Figure 2.4.4 shows this Answer Report.

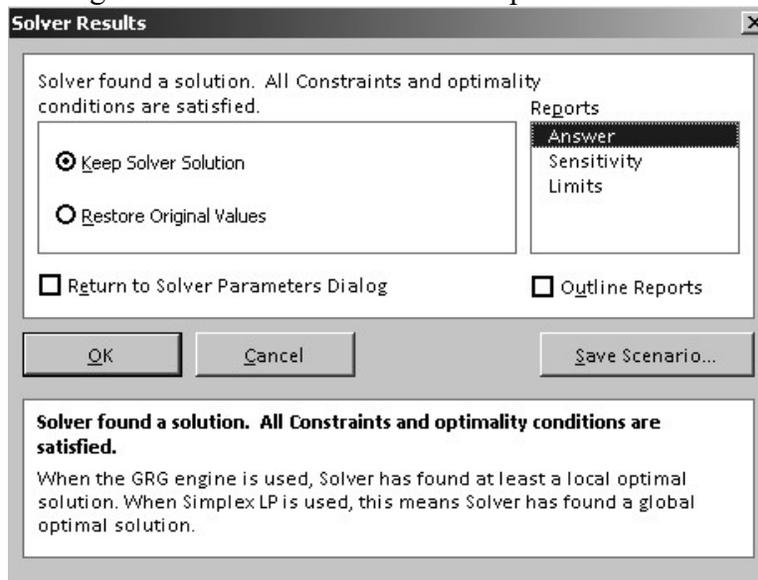


Figure 2.4.3: Generating an Answer Report in Excel Solver

Objective Cell (Max)					
Cell	Name	Original Value	Final Value		
\$J\$8	Objective Function [Profit (\$)] Total Profit	\$0.00	\$6,132.57		

Variable Cells					
Cell	Name	Original Value	Final Value	Integer	
\$B\$6	Decision Values [daily production rate] High Rise (x1)	0	0	Cont	in
\$C\$6	Decision Values [daily production rate] Max-Riser (x2)	0	4.282944345	Cont	in
\$D\$6	Decision Values [daily production rate] Stuff It (x3)	0	45.12172352	Cont	in
\$E\$6	Decision Values [daily production rate] Zoom (x4)	0	72.32746858	Cont	in
\$F\$6	Decision Values [daily production rate] Sprint (x5)	0	104.9019749	Cont	in
\$G\$6	Decision Values [daily production rate] Rocket (x6)	0	89.82118492	Cont	in

Constraints					
Cell	Name	Cell Value	Formula	Status	Slack
\$H\$11	Cutting Time (minutes) Used	420	\$H\$11<=\$J\$11	Binding	0
\$H\$12	Upper Finishing Time (minutes) Used	1260	\$H\$12<=\$J\$12	Binding	0
\$H\$13	Insole Stitching Time (minutes) Used	840	\$H\$13<=\$J\$13	Binding	0
\$H\$14	Molding Time (minutes) Used	2100	\$H\$14<=\$J\$14	Binding	0
\$H\$15	Sole-to-Upper Joining Time (minutes) Used	2100	\$H\$15<=\$J\$15	Binding	0
\$H\$16	Inspecting Time (minutes) Used	799.3421903	\$H\$16<=\$J\$16	Not Binding	40.65780969

**Figure 2.4.4:** Answer Report for Pallas Sport Shoe Company

Notice that the Answer Report is split into three sections: (1) Objective Cell (Max), (2) Variable Cells, and (3) Constraints.

- Q2. Use the “Objective Cell (Max)” section of the Answer Report in Figure 2.4.4 to complete the following.
- Why is “\$J\$8” listed under “Cell?”
  - Why do you think “Objective Function [Profit (\$)] Total Profit” is listed under “Name?”
  - Why do you think the “Original Value” is 0?
  - What is the “Final Value” referring to?
- Q3. Use the “Variable Cells” section of the Answer Report in Figure 2.4.4 to complete the following.
- What are the “Original Value” and “Final Value” columns referring to?
  - Interpret the information given in the “Final Value” column in terms of the problem context.
  - How do you think the production manager should handle the decimal values that appear in the “Final Value” column?
- Q4. Use the “Constraints” section of the Answer Report in Figure 2.4.4 to complete the following.
- Interpret the information given in the “Cell Value” column in terms of the problem context.
  - The first five constraints are binding and the sixth one is not. What does this mean in terms of the problem context?
  - How are the values in the “Slack” column calculated?

- d. If you were only given the slack value for a constraint, how could you determine whether that constraint is binding?
- Q5. Which of the six sport shoe lines should be produced, and at what daily rates, in order to maximize profit? (Approximate to two decimal places.)

Suppose Pallas Sport Shoe Company considers adding another five minutes of cutting time each day. Therefore, the cutting time constraint is changed from 420 to 425. The Answer Report for this new scenario is shown in Figure 2.4.5.

## Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$J\$8	Objective Function [Profit (\$)] Total Profit	\$0.00	\$6,160.47

## Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$6	Decision Values [daily production rate] High Rise (x1)	0	0	Contin
\$C\$6	Decision Values [daily production rate] Max-Riser (x2)	0	6.700179533	Contin
\$D\$6	Decision Values [daily production rate] Stuff It (x3)	0	48.89766607	Contin
\$E\$6	Decision Values [daily production rate] Zoom (x4)	0	74.55655296	Contin
\$F\$6	Decision Values [daily production rate] Sprint (x5)	0	100.4452424	Contin
\$G\$6	Decision Values [daily production rate] Rocket (x6)	0	85.86714542	Contin

## Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$H\$11	Cutting Time (minutes) Used	425	\$H\$11<=\$J\$11	Binding	0
\$H\$12	Upper Finishing Time (minutes) Used	1260	\$H\$12<=\$J\$12	Binding	0
\$H\$13	Insole Stitching Time (minutes) Used	840	\$H\$13<=\$J\$13	Binding	0
\$H\$14	Molding Time (minutes) Used	2100	\$H\$14<=\$J\$14	Binding	0
\$H\$15	Sole-to-Upper Joining Time (minutes) Used	2100	\$H\$15<=\$J\$15	Binding	0
\$H\$16	Inspecting Time (minutes) Used	800.0574506	\$H\$16<=\$J\$16	Not Binding	39.94254937

**Figure 2.4.5:** Answer Report for Pallas Sport Shoe Company with 425-minute Cutting Time constraint

- Q6. How does the Answer Report in Figure 2.4.5 differ from the one in Figure 2.4.4?
- Q7. Is the first constraint still binding? Do you think Pallas Sport Shoe Company should add this extra five minutes of cutting time each day? Explain your reasoning.

Next, suppose Pallas Sport Shoe Company considers subtracting (rather than adding) five minutes of cutting time each day. Therefore, the cutting time constraint is changed from 420 to 415. The Answer Report for this new scenario is shown in Figure 2.4.6.

## Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$J\$8	Objective Function [Profit (\$)] Total Profit	\$0.00	\$6,104.67

## Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$6	Decision Values [daily production rate] High Rise (x1)	0	0	Contin
\$C\$6	Decision Values [daily production rate] Max-Riser (x2)	0	1.865709156	Contin
\$D\$6	Decision Values [daily production rate] Stuff It (x3)	0	41.34578097	Contin
\$E\$6	Decision Values [daily production rate] Zoom (x4)	0	70.0983842	Contin
\$F\$6	Decision Values [daily production rate] Sprint (x5)	0	109.3587074	Contin
\$G\$6	Decision Values [daily production rate] Rocket (x6)	0	93.77522442	Contin

## Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$H\$11	Cutting Time (minutes) Used	415	\$H\$11<=\$J\$11	Binding	0
\$H\$12	Upper Finishing Time (minutes) Used	1260	\$H\$12<=\$J\$12	Binding	0
\$H\$13	Insole Stitching Time (minutes) Used	840	\$H\$13<=\$J\$13	Binding	0
\$H\$14	Molding Time (minutes) Used	2100	\$H\$14<=\$J\$14	Binding	0
\$H\$15	Sole-to-Upper Joining Time (minutes) Used	2100	\$H\$15<=\$J\$15	Binding	0
\$H\$16	Inspecting Time (minutes) Used	798.62693	\$H\$16<=\$J\$16	Not Binding	41.37307002

**Figure 2.4.6:** Answer Report for 415-minute Cutting Time constraint

- Q8. How does the Answer Report in Figure 2.4.6 differ from the one in Figure 2.4.4?
- Q9. Is the first constraint still binding? Do you think Pallas Sport Shoe Company should subtract this five minutes of cutting time each day? Explain your reasoning.

Now, suppose Pallas Sport Shoe Company considers adding another 130 minutes of cutting time each day. Therefore, the cutting time constraint is changed from 420 to 550. The Answer Report for this new scenario is shown in Figure 2.4.7.

## Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$J\$8	Objective Function [Profit (\$)] Total Profit	\$0.00	\$6,781.57

## Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$6	Decision Values [daily production rate] High Rise (x1)	0	0	Contin
\$C\$6	Decision Values [daily production rate] Max-Riser (x2)	0	61.57068063	Contin
\$D\$6	Decision Values [daily production rate] Stuff It (x3)	0	131.9371728	Contin
\$E\$6	Decision Values [daily production rate] Zoom (x4)	0	123.1413613	Contin
\$F\$6	Decision Values [daily production rate] Sprint (x5)	0	0	Contin
\$G\$6	Decision Values [daily production rate] Rocket (x6)	0	0	Contin

## Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$H\$11	Cutting Time (minutes) Used	536.5445026	\$H\$11<=\$J\$11	Not Binding	13.45549738
\$H\$12	Upper Finishing Time (minutes) Used	1260	\$H\$12<=\$J\$12	Binding	0
\$H\$13	Insole Stitching Time (minutes) Used	840	\$H\$13<=\$J\$13	Binding	0
\$H\$14	Molding Time (minutes) Used	2093.403141	\$H\$14<=\$J\$14	Not Binding	6.596858639
\$H\$15	Sole-to-Upper Joining Time (minutes) Used	2100	\$H\$15<=\$J\$15	Binding	0
\$H\$16	Inspecting Time (minutes) Used	818.0104712	\$H\$16<=\$J\$16	Not Binding	21.9895288

**Figure 2.4.7:** Answer Report for 550-minute Cutting Time constraint

- Q10. How does the Answer Report in Figure 2.4.7 differ from the one in Figure 2.4.4?
- Q11. Do you think Pallas Sport Shoe Company should add this extra 130 minutes of cutting time each day? Explain your reasoning.

Recall that the decision variable  $x_1$  is not in the optimal solution. A logical question to ask is whether increasing the profitability of  $x_1$  could allow it to enter the optimal solution and, if so, how much of an increase would be necessary. Return again to the spreadsheet in Figure 2.4.2, reset the cutting time constraint to its original value of 420 minutes, and change the value of the objective function coefficient of  $x_1$  from 18 to 19. The Answer Report for this new scenario is shown in Figure 2.4.8.

- Q12. In terms of the problem, what does this change represent?

## Objective Cell (Max)

Cell	Name	Original Value	Final Value
\$J\$8	Objective Function [Profit (\$)] Total Profit	\$0.00	\$6,216.90

## Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$6	Decision Values [daily production rate] High Rise (x1)	0	170.6951872	Contin
\$C\$6	Decision Values [daily production rate] Max-Riser (x2)	0	8.983957219	Contin
\$D\$6	Decision Values [daily production rate] Stuff It (x3)	0	125.7754011	Contin
\$E\$6	Decision Values [daily production rate] Zoom (x4)	0	0	Contin
\$F\$6	Decision Values [daily production rate] Sprint (x5)	0	0	Contin
\$G\$6	Decision Values [daily production rate] Rocket (x6)	0	0	Contin

## Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$H\$11	Cutting Time (minutes) Used	420	\$H\$11<=\$J\$11	Binding	0
\$H\$12	Upper Finishing Time (minutes) Used	1260	\$H\$12<=\$J\$12	Binding	0
\$H\$13	Insole Stitching Time (minutes) Used	716.4705882	\$H\$13<=\$J\$13	Not Binding	123.5294118
\$H\$14	Molding Time (minutes) Used	1873.15508	\$H\$14<=\$J\$14	Not Binding	226.8449198
\$H\$15	Sole-to-Upper Joining Time (minutes) Used	2100	\$H\$15<=\$J\$15	Binding	0
\$H\$16	Inspecting Time (minutes) Used	619.8930481	\$H\$16<=\$J\$16	Not Binding	220.1069519

**Figure 2.4.8:** Answer Report for \$19 High Rise shoe profit

- Q13. How does the Answer Report in Figure 2.4.8 differ from the one in Figure 2.4.4?
- Q14. Why may Pallas Sport Shoe Company not want this new optimal solution?
- Q15. Explain, in your own words, the usefulness of Answer Reports.

## Chapter 2 (LP Maximization) Homework Questions

1. Anderson Cell Phone Company has started cell phone production. It produces smart phones and standard phones. Initially, they hired 10 workers for the assembly line. The workers are paid for 8 hours per day. However, they spend only 7 hours assembling the phones because of a 30-minute lunch break and two 15-minute breaks. A smart phone takes 2.5 minutes to assemble and a standard phone takes 1.5 minutes to assemble. The company receives a delivery of 2000 LCD screens per day from its supplier. Profit margins for a smart phone and a standard phone are \$40 and \$30, respectively. Anderson Company is interested in determining the product mix that gives them the highest daily profit.
  - a. Define the decision variables for the problem.
  - b. Use the decision variables to define the objective function.
  - c. Use decision variables to define the assembly constraint.
  - d. Use decision variables to define the screen constraint.
  - e. Graph the feasible region.
  - f. Pick two points in the feasible region and calculate the profit for each point.
  - g. What are the corner points of the feasible region?
  - h. Calculate the profit at each corner point.
  - i. Which point gives the highest profit? What is the optimal product mix?
  - j. Find the optimal production plan with Excel Solver to verify the optimal solution.
  
2. GA Sports makes professional and regular soccer balls. Typically, soccer balls are made up of 4 elements: the cover, the stitching, the lining, and the bladder. GA Sports uses a synthetic leather cover for professional balls and puts 4 layers of cotton linings underneath. It uses a rubber cover for regular balls and puts 3 layers of cotton linings underneath. The cover and the linings are created by cutting large pieces of each material into 32 panels for a professional soccer ball and 18 panels for a regular soccer ball. It takes 14 minutes to carefully cut the casing for a professional ball. It takes only 7 minutes to cut the material for a regular ball. GA Sports has one experienced cutter who works 7 hours per day. The 32 panels of a professional soccer ball are stitched together. An experienced worker can stitch 4 balls in a workday. The company has four experienced stitchers. The regular balls are thermally molded on a special machine in twelve minutes. The company has one machine that is run by a machinist for six hours per day. The company makes \$15 profit on professional balls and \$10 on regular balls.
  - a. Define the decision variables for the problem.

- b. Use the decision variables to define the objective function.
  - c. Use decision variables to define the cutting time constraint.
  - d. Use decision variables to define the stitching time constraint.
  - e. Formulate the problem.
  - f. Find the feasible region graphically.
  - g. Find the optimal production plan using MS Excel Solver.
3. Family Cow is a small dairy. The owner is the only person who works in the dairy; he works 8 hours per day. He makes and sells cream and butter and donates the skim milk to a charity nearby. He is able to make 1 quart of cream from 10 quarts of milk or 1 pound of butter from 20 quarts of milk. It takes 20 minutes to get a quart of cream and 30 minutes to make a pound of butter. The dairy has 300 quarts of milk daily. Family Cow's prices are higher because the products are organic: a quart of organic cream is sold for \$7 and a pound of organic butter for \$12. He can only sell eight pounds of butter per day to a local store.
- a. Define the decision variables for the problem.
  - b. Use the decision variables to define the objective function.
  - c. Formulate the problem.
  - d. Determine the optimal solution graphically.
  - e. Find the optimal production plan using MS Excel Solver.
4. The owner of The Family Cow dairy has been using the optimal production plan for several months. He is considering adding plain yogurt and cheese to the products he sells. It takes 10 quarts of milk to produce 7 lbs. of yogurt or 3 lbs. of cheese. Prices of yogurt and cheese are \$3.5/lb. and \$5.5/lb., respectively. It takes 10 minutes to make a pound of yogurt and 15 minutes for a pound of cheese.
- a. Define the additional decision variables.
  - b. Formulate the problem.
  - c. Find the new optimal product mix and the daily profit using MS Excel Solver. How many different products does he make? How much more money will he make by offering two more products? Would you recommend he change his production plan?

5. Katia has won \$200,000 from the lottery. She considers investing the money in a bond fund and a domestic stock fund. The projected annual return for the bond fund is 8% and for the stock fund is 15%. Her friend is experienced with investments. She suggests she invest at most \$75,000 in the stock fund. However, the stock and bond funds could go down in value. The brokerage firm told Katia that the most she could lose in the next year is 5% of her original investment in the bond fund and 20% of her original investment in the stock fund. She wants to limit her total potential losses to no more than \$20,000. How should she invest her winnings?
- Define the decision variables for the problem.
  - Formulate the problem.
  - Determine the optimal investment plan using MS Excel Solver.
6. In addition to the bond fund and the domestic stock fund, Katia also considers investing in an international stock fund. The projected annual return for that stock fund is 22%. The maximum investment in any stocks is still \$75,000. The most she could lose in the next year in this international stock is 25%.
- What is her optimal investment strategy?
  - Will she make more money than before?
7. John Farmer is studying operations research in school. He is curious about applying what he learns in class to actual problems. John's father owns a farm in Missouri and has 640 acres under cultivation. John would like to help his father determine the mix of corn and soybeans to plant that would maximize his father's profit. Table 2.4.1 contains some data that John collected about corn and soybean production in Missouri.

	<b>Corn</b>	<b>Soybeans</b>
<b>Price/Bushel</b>	\$2.90-3.50	\$7.85-8.85
<b>Yield/Acre</b>	155.8 bu.	41.4 bu.
<b>Seed Cost/Acre</b>	\$45.50	\$34.00
<b>Fertilizer cost/acre</b>	\$88.10	\$37.80
<b>Fuel Cost/Acre</b>	\$24.40	\$10.00
<b>Worker Cost/Acre</b>	\$13.00	\$9.50

**Table 1:** 2007-08 USDA corn and soybean estimates for Missouri

- In order to formulate the problem, John must know the price per bushel at which corn and soybeans can be sold. However, the USDA data contains a range of values for both of these crops. What value do you think John should use for each? Why?
- What is the largest gross revenue before considering expenses that Mr. Farmer can make? What crop mix produces this gross revenue?

- c. What is the largest net revenue after expenses that Mr. Farmer can make? What crop mix produces this net revenue?
- d. Suppose Mr. Farmer has budgeted only \$60,000 to cover all of the expenses of his crop production. What is the largest net revenue he can earn under this constraint?
8. Corn and soybeans are used in the production of biofuels. Biofuel consumption is important for the environment, because greenhouse gas emissions are reduced 12% by ethanol combustion and 41% by biodiesel combustion. The total corn and soybean production in the United States can meet only 12% of the demand for gasoline and 6% of the demand for diesel fuel. In order to encourage corn production, the USDA pays a subsidy to increase the price per bushel to \$3.50. However, to get the subsidy, the farmer must produce at least 40,000 bushels of corn.
- a. Assuming that the price per bushel that Mr. Farmer can sell his corn for without the subsidy is \$2.90, should Mr. Farmer accept the constraint of producing at least 40,000 bushels of corn in order to earn the subsidy?
9. After doing some research, John learned that soybean followed by corn a year after, increases corn yield by 7.5% and saves 25% of the soybean residue nitrogen, which will reduce the fertilizer use in corn production by \$2.50 per acre. The reduction in fertilizer use will also reduce its harmful effect to the environment. In the news, he heard that nitrate leaching causes surface and ground water to degrade. This will harm the living things that use water for drinking or swimming. For all of these reasons, John wants to convince his father, who planted 200 acres of land with soybean last year, to begin to rotate corn and soybeans. Add a constraint that the maximum amount of corn is equal to these 200 acres.
- a. Should John's father begin to rotate his corn and soybean crops by increasing his corn planting?
10. John gathered some data related to wheat production (Table 2.4.2). Go back to the original formulation stated in question 7. Is it profitable to produce some wheat? Use the midpoint of price range (\$3.95) to determine net revenue per acre

	<b>Corn</b>	<b>Soybean</b>	<b>Wheat</b>
<b>Price/bushel</b>	\$2.90-3.50	\$7.85-8.85	\$3.75-4.15
<b>Yield/acre</b>	155.8 bushels	41.4 bushels	60 bushels
<b>Seed cost/acre</b>	\$45.50	\$34	\$24
<b>Fertilizer cost/acre</b>	\$88.10	\$37.80	\$69.25
<b>Fuel cost/acre</b>	\$24.40	\$10	\$40
<b>Worker cost/acre</b>	\$13	\$9.50	\$9

**Table 2:** 2007-2008 Corn, soybean, and wheat estimates for Missouri

11. At the end of Problem 2.1, you were asked to formulate the Computer Flips problem after the addition of two new products. Your formulation should have used four decision variables and should have included eight constraints, including four non-negativity

constraints. In an Excel spreadsheet, set up this formulation, and use Solver to obtain a solution.

- a. What is the optimal solution?
  - b. What does this optimal product mix imply about the planned addition of new products?
  - c. What research do you think the Sales Department should conduct before implementing the optimal product mix?
12. Elegant Fragrances, Ltd. decides to produce two new perfumes, *L'Arbre d'Amour* and *Evening Rose*. The factory management asked the industrial engineering department to develop a mathematical model for maximizing the profit obtained from producing these products. There are also some limitations in resources, budget and the capacity of the factory that should be considered in the model. At the beginning, the industrial engineering team studied the problem and gathered information from which a model can be developed. The team collected the following information:
- Each perfume is made of two main components: a fragrant perfume oil and a solvent. The solvent, such as a combination of ethanol and water, is necessary to reduce the allergic reactions of skin to the perfume oil. The solvent is a large percentage of the overall final product.
  - A fragrant oil for *L'Arbre d'Amour* is obtained from Mango Pulp, Tea Leaves, and Juniper Berry, and a fragrant oil for *Evening Rose* is obtained from Mango Pulp, Tea Leaves, and White Rose.
  - There are two main processes in the production of these perfumes: extraction and blending. In the extraction stage, physical and chemical processes change the raw materials and the perfume oil is extracted. In the blending stage, the perfume oil is blended with the solvent. However, Elegant Fragrances does not work directly with the fragrance raw materials, but instead purchases fragrance essences from suppliers and only blends them.

		<i>L'Arbre d'Amour</i>	<i>Evening Rose</i>	
<b>Income (per pound)</b>		\$370	\$215	
<b>Percentage of raw materials</b>	<b>Fragrance Oil (Pure)</b>	<b>Mango</b>	3%	5%
		<b>Tea Leaves</b>	5%	7%
		<b>Juniper Berry</b>	7%	0
		<b>White Rose</b>	0	8%
	<b>Solvent</b>	<b>Ethanol and Water</b>	85%	80%
<b>Process Costs (Per Pound)</b>		<b>Blending</b>	\$12	\$12

**Table 3:** Some information about the perfumes

	Availability (pounds per year)	Cost (\$ per pounds)
<b>Mango</b>	1400	14
<b>Tea Leaves</b>	1600	10
<b>Juniper Berry</b>	1000	384
<b>White Rose</b>	1500	56
<b>Ethanol and Water</b>	Unlimited	7

**Table 4:** Ingredient information

The total cost for making each perfume is the \$12 processing cost plus the cost of ingredients. The cost for ingredients to make *L'Arbre d'Amou* is determined by multiplying the percentage of each ingredient by the corresponding costs.

$$\text{Ingredient cost} = .03(14) + .05(10) + .07(384) + 0(56) + .85(7) = \$33.75$$

$$\text{Net Profit} = 370.00 - 12 - 33.75 = \$324.25$$

- a. Determine the net profit for *Evening Rose*.
  - b. Define the decision variables as the amount of annual production of each perfume and help the industrial engineering team formulate this problem for maximizing the profit within the given constraints.
  - c. Use Solver to obtain the optimal solution for the Elegant Fragrances problem.
13. The management at Elegant Fragrances is considering producing two new perfumes, *Evergreen* and *Embrasser du Soir*. The income and costs, as well as the key ingredients and their proportions for the new perfumes are given in Table 2.4.5 below. If the availability of resources is unchanged, what are the optimal production rates for each of the four perfumes so as to maximize profit?

			<i>Evergreen</i>	<i>Embrasser du Soir</i>
<b>Income (per pound)</b>			\$490	\$235
<b>Percentage of raw materials</b>	<b>Fragrance Oil (Pure)</b>	<b>Mango</b>	8%	7%
		<b>Tea Leaves</b>	7%	5%
		<b>Juniper Berry</b>	9%	0
		<b>White Rose</b>	0	10%
	<b>Solvent</b>	<b>Ethanol and Water</b>	76%	78%
<b>Process Costs (Per Pound)</b>		<b>Blending</b>	\$12	\$12

**Table 5:** New Perfumes Information

## Chapter 2 Summary

### What have we learned?

Linear programming is a process of taking a real world situation, modeling it with inequalities, and finding the best or optimal solution.

- Modeling the situation
  - We start by finding the decision variables – what things can you choose? Typically this is how much to make of a particular product or how much to invest in a particular option. We assign a variable for each of these choices.
  - Next we write an objective function that captures the goal of the problem. – What will determine when you have found the optimal solution? This is the equation that we want to maximize.
  - Finally we define the constraints - those things that limit our choices. These are typically the amount of money, time, people, or resources available.
- Once we have defined our problem, we use a spreadsheet program such as Microsoft Excel to find the optimal solution. After entering the inequalities we set up the Solver parameters and run Solver. This gives us an answer and sensitivity report.
- The answer report will show:
  - The objective function's final value
  - The value for each of the decision variables
  - The amount of each constraint that is used

## Terms

<b>Answer Report</b>	A report that details the optimal solution, lists whether constraints are binding or non-binding, and gives the slack for each constraint
<b>Binding Constraint</b>	A constraint that is satisfied as a strict equality in the optimal solution; all of the available constraint is used
<b>Constraint</b>	A condition that must be satisfied, represented by equations or inequalities
<b>Decision Variable</b>	A quantity that the decision-maker controls
<b>Feasible Solution</b>	A solution that satisfies all the constraints
<b>Final Value</b>	A column in the Answer and Sensitivity Reports that refers to the number in the decision variable cells <i>after</i> you use Solver (i.e., the decision variable and objective function values for the optimal solution)
<b>Line of Constant Profit</b>	A line representing the objective function, where every point on the line generates the same profit
<b>Linear Programming</b>	A mathematical technique for finding the optimal value of a linear objective function subject to linear constraints when the decision variables can take on fractional values.
<b>Mathematical Programming</b>	A mathematical approach to allocating limited resources among options in an optimal manner (includes linear programming, integer programming, and binary programming)
<b>Non-Binding Constraint</b>	A constraint for which all available resources are not used
<b>Objective Function</b>	The function that is to be optimized
<b>Optimal Solution</b>	The feasible solution with the best value for the objective function
<b>Original Value</b>	A column in the Answer Report that refers to the number in the decision variable cells <i>before</i> you use Solver
<b>Product Mix</b>	The composition of all goods and/or services being produced

**Production Rate**      The number of products made in a given period of time

**Slack**                      The amount of a resource that is not used in the optimal solution

## Chapter 2 (LP Maximization) Objectives

### You should be able to:

- Identify the decision variables
- Define the objective function by finding the goal to be solved for the situation
- Identify the constraints and write inequalities to model them.
- Enter each of these into Microsoft Excel
- Use Solver to find the Optimal Solution and generate Answer and Sensitivity Reports
- Analyze the Answer Report

## Troubleshooting

### What can go wrong?

Troubleshooting is a valuable skill when using Excel and Solver. Quite often a problem is developed and solved and the answer will not make sense. As you work through this book try to remember the mistakes you make so you can avoid them in the future.

- One thing that students may do when working through the problems with the text is to look ahead. They realize that the answers to the questions are on the next page and will copy them onto their spreadsheet. This can lead to problems running Solver. When setting up the constraints, the right side must be a value – how much of the resource is available. The left side must be a formula that multiplies the decision variables by the amount used for each one. This formula does not appear on the screen, only the value appears. Typing values printed in the book rather than the formula will cause a problem.
- Another common mistake is to forget to change the direction of the inequality in Solver. The default in Solver is a less than or equal ( $\leq$ ) constraint. For example, if you have up to 40 hours to produce something, you use the  $\leq$  constraint. Some situations require something to be at least a certain value. For example if you make tables and chairs you have to make at least 4 chairs for each table. This requires you to use the greater than or equal ( $\geq$ ) constraint.
- When a model changes, don't forget to make the changes in Solver. Recall that when we develop a spreadsheet, we start by entering our decision variables, then write our objective function, and finally a row for each constraint. We then start Solver and enter the decision variables, objective function and constraints. If you revise a problem, either by adding new decision variables or adding new constraints, you follow the same process. First change the decision variables and/or constraints in Excel but then you must make the same changes in Solver. Otherwise the computer will not recognize that the situation has changed.

## Chapter 2 Study Guide

1. What are decision variables? Where do they come from in the word problem?
2. What is the objective function? Where does it come from in the word problem?
3. What are constraints? Where do they come from in the word problem?
4. What information have we used from the Answer Reports?
5. Write a definition for each in your own words.
  - a. Binding constraint-
  
  - b. Non-binding constraint-
6. What is the “Final Value” on an Answer Report?
7. What is slack? What does the cell value tell us?

## References

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## Appendix A: Using Excel Solver in Microsoft Office 2003 and 2007

The majority of the steps for using Excel Solver are the same in all versions of Microsoft Office. However, there are a few differences.

### Microsoft Office 2003

To add in Solver in Microsoft Office 2003, go to Add-Ins under the Tools menu and click on it. The Add-Ins window will appear. Then, check the Solver Add-In box and then click OK.

Next, set up the spreadsheet in the same way as detailed in Steps 1-4 of Section 2.2.6.

Once the spreadsheet is properly set up, click on the cell containing the objective function and choose Solver from the Tools menu. A Solver Parameters window will open, as shown in Figure

	A	B	C	D	E	F	G	H
1	SK8MAN, INC.							
2	Maximization Problem							
3								
4	Decision Variable	x1	x2	x3				
5	Decision Variable Values							
6								
7	Objective Function	15	35	20			0	
8	Shaping time constraint	5	15	4	0	<=	2400	
9	Truck availability constraint	2	2	2	0	<=	700	
10	North American maple constraint	0	7	0	0	<=	840	
11	Chinese maple constraint	7	0	7	0	<=	1470	
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								

The Solver Parameters dialog box is open, showing the following settings:

- Set Target Cell:  $G7$
- Equal To:  Max  Min  Value of: 0
- By Changing Cells: (empty)
- Subject to the Constraints: (empty)

2.A.1.

**Figure 2.A.1:** Solver Parameters window in Microsoft Office 2003

Verify that the “Max” circle is filled in. Then fill in the “By Changing Cells” and “Subject to the Constraints” windows as described in Steps 7-8 in Section 2.2.6

To include the non-negativity constraint, choose Options and check the box that says “Assume Non-Negative.” Also, check the box that says “Assume Linear Model.” See Figure 2.A.2 for an illustration of this.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H
1	SK8MAN, INC.							
2	Maximization Problem							
3								
4	Decision Variable	x1	x2	x3				
5	Decision Variable Values	0	0	0				
6								
7	Objective Function	15	35	20			0	
8	Shaping time constraint	5	15	4	0	<=	2400	
9	Truck availability constraint	2	2	2	0	<=	700	
10	North American maple constraint	0	7	0	0	<=	840	
11	Chinese maple constraint	7	0	7	0	<=	1470	

The Solver Options dialog box is open, showing the following settings:

- Max Time: 100 seconds
- Iterations: 100
- Precision: 0.000001
- Tolerance: 5%
- Convergence: 0.0001
- Assume Linear Model
- Use Automatic Scaling
- Assume Non-Negative
- Show Iteration Results
- Estimates:  Tangent,  Quadratic
- Derivatives:  Forward,  Central
- Search:  Newton,  Conjugate

**Figure 2.A.2:** Solver Options window in Microsoft 2003

Finally, click “Solve” and choose the desired reports, as shown in Figure 2.A.3.

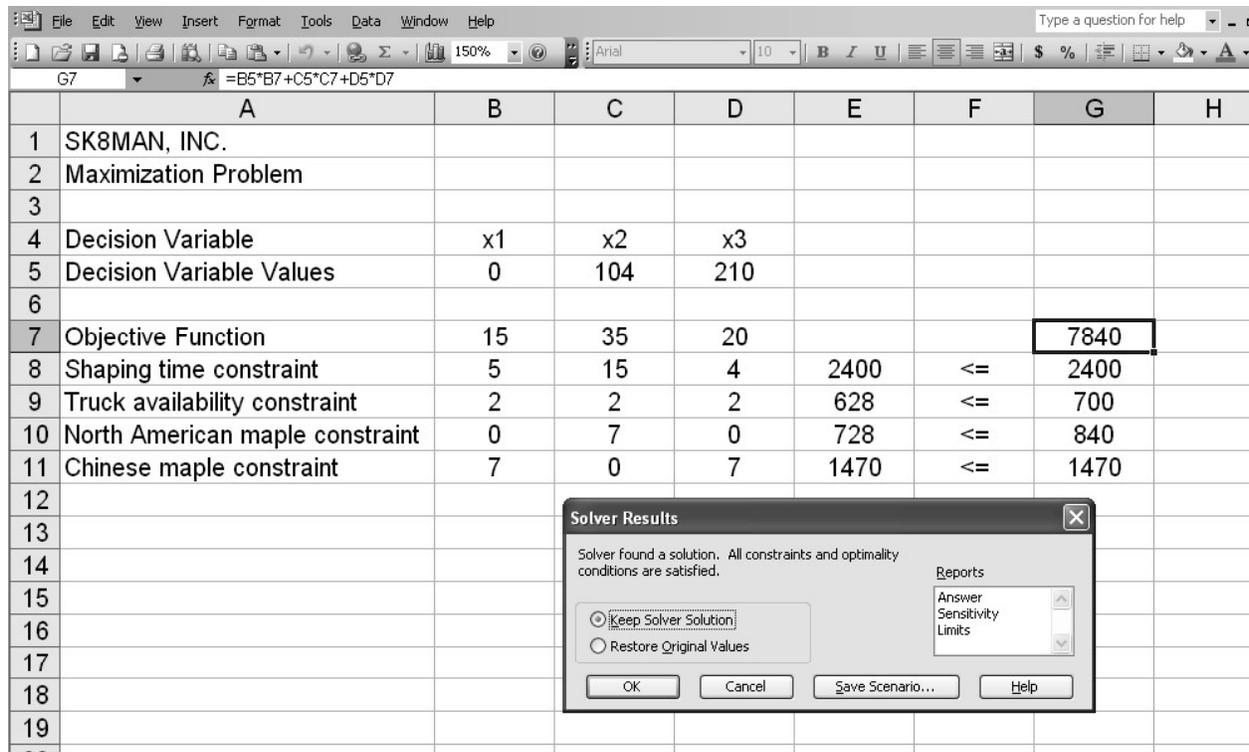


Figure 2.A.3: Solver results window

## Microsoft Office 2007

To add in Solver in Microsoft Office 2007, click the Microsoft Office Button and choose “Excel Options.” Click “Add-Ins.” Under the “Manage” box, select “Excel Add-ins” and click “Go.” Check the “Solver Add-in” box and click “OK.”

Once Solver has been added, the Solver command is in the Analysis group on the Data tab, as shown in Figure 2.2.9.

All other steps for using Excel Solver in Microsoft Office 2007 are the same as for Microsoft Office 2003, given above.

