Fuel Economy

Which Measure to Use?

Mathematical Goals

The student will investigate different rates determining which units of measure are more meaningful in a given situation.

The student will:

- Read a scenario and use rates to solve multi-step problems.
- Understand there are two unit rates associated with each ratio.
- Calculate rates
- Compare rates
- Work with rates in meaningful contexts.

Before the lesson, practice mental mathematics (5-10 minutes).

Number Talk Possibilities: Select two or three depending on student abilities.

- If your car gets 20 mpg and you travel for 100 miles, how many gallons of gas do you use?
- Convert 25 mpg to gallons per 100 miles.
- Convert 5 gallons per 100 miles traveled to miles per gallon.
- Find the percent increase if mpg increase from 20 mpg to 25 mpg.
- Find the percent decrease from 28 mpg to 21 mpg.

Fuel Economy

Rate is used in many contexts to describe the ratio of two related measures. These two measures may have different units or similar units of measurement. A common example of different units of measurement is the ratio of distance to time to describe the speed of travel: miles *per* hour is a common example of a rate that compares two measures having different units. An example of a rate whose numerator and denominator has similar units of measurement is one involving disease. These rates are reported as the number of sick people over 1,000 people. Both the numerator and denominator are people. Usually we say that this is the rate of sick people *per* 1,000 people.

The concept of rate is used in many situations. Some thought should be given to the units of measurement. This activity discusses fuel economy for cars. The standard measure in the U.S. is miles per gallon. This example shows that a 25% improvement in miles per gallon does not result in using 25% less gas. The example suggests a better measure is gallons per hundred miles driven.

Sergei Ellison and Laura Brin are friends. They are comparing notes as they both look to replace their old cars with more fuel efficient models. Laura drives a ten year-old SUV that gets only 20 mpg. Sergei drives a seven year old compact car that gets 28 mpg. The SUV Laura plans to buy will get an estimated 25 mpg. The car Sergei will buy gets 35 mpg.

"You've got to admit," Sergei bragged, "that I'll be improving my fuel economy more than you will. I'll definitely save more money."

"Not so fast," replied Laura with a laugh. Laura was better at math than Sergei. I will save more money per month."

"No way," Sergei argued.
"Tell you what," Laura said. "I'm ready to bet you a dinner for two at a fancy restaurant that I'm saving more money than you."

"It's a bet," Sergei said. "I'm going to start saving my appetite right now!"

"Okay," Laura said, "each of us will start with a full tank of gasoline in our current vehicle. We'll drive as usual and buy gasoline as needed. When we have driven exactly 1,000 miles, each of us will fill up one last time. Next, we'll do the same for each new vehicle. Then we will compare how much each of saves with our newer higher mileage vehicle.

"I can't believe you're actually making this bet," Sergei said confidently. "Remember, no taking back the bet!"

1. How many more miles per gallon would Sergei get with his new vehicle? How many more miles per gallon would Laura get with her new vehicle?

2. What was the percentage increase for each person?

Gasoline was selling for $2.50 during the experiment. At the end of the 1,000-mile experiment, they each totaled all their bills. The totals were compared. Laura saved $25.00. Sergei saved only $17.86. Sergei was shocked. “How did that happen?!?” he exclaimed.

Now, let’s work out the details of the experiment.

3. How many gallons did Laura use to drive 1,000 miles in her old SUV?

4. How much money did she spend on gasoline at $2.50 per gallon?

5. How many gallons did Laura use to drive her newer SUV 1,000 miles?

6. How much money did she spend on gasoline at $2.50 per gallon?

7. What was Laura’s percent savings? How does it compare to the percent improvement in fuel economy?

Now repeat the calculations for Sergei.

8. With his old car, how many gallons did Sergei use and what did it cost?

9. With his newer car, how many gallons did Sergei use and what did it cost?

10. What was Sergei's percent savings? How does it compare to the percent improvement in fuel economy?

The next night the two friends ate dinner at the exclusive Russian Tea Room. Sergei asked for an explanation. "Did you do these financial calculations in your head? Is that how you figured out you would save more money?" Laura answered, "There is a short cut. All other countries in the world report fuel economy as the inverse (gallons/miles versus miles/gallons). Their fuel economy measure is the number of gallons per hundred miles (or kilometers) of travel. With this measure it was easy for me to see I would save more money."

Let's do the calculations. Twenty miles per gallon means it takes 5 gallons to travel 100 miles

$$\frac{100\ miles}{20\ miles/gallon} = 5\ gallons\ (per\ 100\ miles)$$

The corresponding calculation for the newer SUV is

$$\frac{100\ miles}{25\ miles/gallon} = 4\ gallons\ (per\ 100\ miles)$$

This means the newer SUV would save 1 gallon. This is equal to a 20% savings.

11. How does this compare to the answer found in question 7?

The total dollar savings is $10 \times \$2.50 = \$25.00$.

Now do the same calculations for Sergei's vehicle.

12. What are the gallons per 100 miles for Sergei's old and new vehicle?

13. How much gasoline would he save driving 100 miles? 1,000 miles?

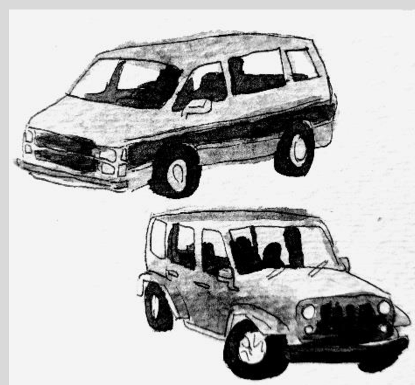
14. What is the percent savings in gasoline? How does this compare to the answer found in question 10?

15. How much money did Sergei save?

Sergei's gasoline savings (7mpg) was higher than Laura's (5mpg). Yet, the amount of gasoline he saved was less than Laura's.

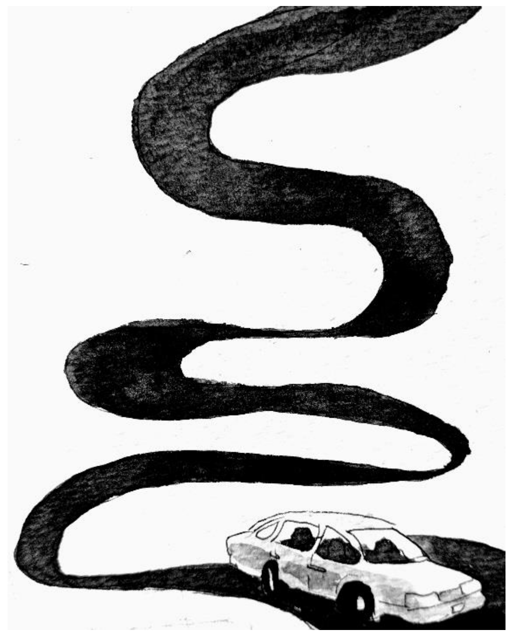
16. Explain why this is true.

Project Idea: Select two vehicles and research expected miles per gallon for city driving on each. If both vehicles promise a 5 mpg increase in the next model year, and gas prices remain relatively constant (say $2.75 per gallon), which vehicle will be impacted the most? Justify your decision by showing calculations for fuel economy and cost savings.



Practice Problems - Fuel Economy

In the above example, an increase from 20 mpg to 25 mpg saved more money than the improvement from 28 mpg to 35 mpg. Auto executives would like the government to adopt gallons per 100 miles as the better way to report fuel economy. This way it would be clearer that there is diminishing returns from improvements as cars become more fuel efficient.



Car companies are required to use a common standard driving routine to measure fuel economy. This number is then placed on the car's sticker. However, this number may not match the individual's driving experience. Suppose Theresa bought a fuel efficient car rated as 40 mpg. She kept track of her driving and gas purchases. She found she only averaged 36 mpg. Her mother, Shana, drives a minivan. It is rated as 25 mpg. Shana found she only averaged 23 mpg.

1. Which driver do you think would be more upset? Why?

2. What was the absolute difference in mpg for each vehicle?

3. What was the percentage difference for each vehicle?

Both Theresa and Shana drive approximately 1,000 miles each month.

4. How many more gallons of gas will Theresa buy because the rating was not accurate?

5. How many more gallons of gas will Shana buy because the rating was not accurate?

6. Who is more affected by the incorrect rating?

Thinking Through a Lesson Protocol

UNIT: Rates

Suggested Time: One class period

Materials needed: Calculator

Standards

6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio a:b with b not equal to 0 and use rate language in the context of a ratio relationship.

6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.A.3B: Solve unit rate problems including those involving unit pricing and constant speed.

Mathematical Practices:

MP1: Make sense of problems and persevere in solving them.

MP2: Reason abstractly and quantitatively.

MP6: Attend to precision.

MP7: Look for and make use of structure.

Setting up the Problem - Launch	
Selecting tasks/goal setting	(10 minutes) What are the variables that impact an improvement in fuel economy? Ask students to read for five minutes and take notes while reading the article from the Union of Concerned Scientists found at https://www.ucsusa.org/clean-vehicles/fuel-efficiency/fuel-economy-basics.html#.W9nIY9VKjIU which illustrates improvements in fuel economy through 2025. Discuss findings.

Monitoring student work - Explore		
Part I: All Students		
Strategies and misconceptions - Anticipating	**Who - Selecting and sequencing**	**Questions and Statements - Monitoring**
(10 minutes) Read Fuel Economy and answer questions #1 and #2.		Be sure that students have calculated percent increase correctly.
(15 minutes) Have students answer questions #3-#7. Share whole group.		It is imperative that students understand the solution to #7 and the whole group discuss what it means.
(15 minutes) Have students answer questions #8-#10. Share whole group.		It is imperative that students understand the solution to #10 and the whole group discuss what it means.
(20 minutes) Continue reading text and answer questions #11 - #15.		Discuss results for question #14.
(5 minutes) Hold an entire class discussion on #16. Discuss with a partner first, then extend to small groups, then look at it as an entire class.		Don't let students struggle with this concept without some support. It may be difficult for students to understand.

Monitoring individual student work - Explore		
Part II: Specific Groups of Students		
Strategies and misconceptions - Anticipating	**Who - Selecting and sequencing**	**Questions and Statements - Monitoring**
For off-task students or for students that seem to be self-conscious about you listening to them share.		I am just listening or looking to find out how you are working on the problem. This helps me think about what we will do later.
For students that appear to be stuck. Also for when you are having a difficult time understanding their strategies.		Can you tell me a little about your reading? How would you describe the problem in your own words? What facts do you have? Could you try it with simpler numbers?
For students that want to ask you questions, these are ways to uncover their thinking and judge to what extent you want to respond.		Tell me what you've thought about so far. What do you know? Why are you interested in more information about that? Let me say a little about that part.

Managing the discussion – Summarize	
Parts of discussion - Connecting	**Questions and statements - Connecting**
Launching the discussion: Select the problems that students are struggling with or you wish to share out.	Will team 1 start us off by sharing one way of working on this problem? Please raise your hand when you are ready to share your solution. What did you do first when you were working on this problem? Let's start by clearing up a few things about the problem. Let's list some key parts in this problem. What was unclear in the problem?
Eliciting and uncovering student strategies	Joe would you be willing to start us off? What have you found so far? Can you repeat that? Can you explain how you got that answer? How do you know? Walk us through your steps. Where did you begin? Can you show us?
Focusing on Mathematical Ideas	Can you explain why this is true? Does this method always work? How is Bob's method similar to Kelly's method? What do all the solutions have in common? What would happen if I changed the numbers to _____?
Encouraging Interactions	Do you agree or disagree with Kahlil's idea? What do others think? Would someone be willing to repeat what Tom just said? Would anyone be willing to add on to what Sue just said?
Concluding the Discussion	Can anyone tell me some of the big ideas that we learned today? How would you explain what we learned today to a 5^{th} grader? Some of the key points from our discussion today are . . . Tomorrow we will continue our exploration of ______ beginning with the idea from today that ___________.
Post Lesson Notes	You may wish to assign the practice problems that you feel would benefit the students.

Solutions to Text Problems for Fuel Economy

1. How many more miles per gallon would Sergei get with his new vehicle? How many more miles per gallon would Laura get with her new vehicle?

   Sergei increases from 28 mpg to 35 mpg or an increase of 7 mpg. Laura increases from 20 mpg to 25 mpg or an increase of 5 mpg.

2. What was the percentage increase for each person?

   Sergei: 28 mpg to 35 mpg, increases 7 mpg/28 mpg for a 25% increase
   Laura: 20 mpg to 25 mpg, increases 5 mpg/20 mpg for a 25% increase

3. How many gallons did Laura use to drive 1,000 miles in her old SUV?

   20 miles /1 gallons = 1000 miles/50 gallons
   ***OR** 1000 miles/20 miles per gallon = 50 gallons*

4. How much money did she spend on gasoline at $2.50 per gallon?

   $2.50/gallon times 50 gallons is $125.00 per 1000 miles driven

5. How many gallons did Laura use to drive her newer SUV 1,000 miles?

   25 miles /1 gallon = 1000 miles /40 gallons
   ***OR** 1000 miles/25 miles per gallon = 40 gallons*

6. How much money did she spend on gasoline at $2.50 per gallon?

   $2.50/gallon times 40 gallons is $100.00 per 1000 miles driven

7. What was Laura's percent savings? How does it compare to the percent improvement in fuel economy?

   Laura saved $25 out of the original $125 for an increase of 25/125 = 0.20 or 20%.
   This is 5% less than the improvement in fuel economy.

8. With his old car, how many gallons did Sergei use and what did it cost?

   1000 miles/28 miles per gallon = 35.7 gallons
   $2.50/gallon times 35.7 gallons is $89.25 per 1000 miles driven

9. With his newer car, how many gallons did Sergei use and what did it cost?

   1000 miles/35 miles per gallon = 28.6 gallons
   $2.50/gallon times 28.6 gallons = $71.50 per 100 miles driven

10. What was Sergei's percent savings? How does it compare to the percent improvement in fuel economy?

    Sergei saved $89.25 - $71.50 = $17.75. $17.75 out of the original $89.25 for an increase of 17.75/89.25 = 0.1988 or approximately 20%.

    This is 5% less than the improvement in fuel economy.

11. How does this compare to the answer found in question 7?

    The percent savings for Laura and Sergei are approximately the same (20%).

12. What are the gallons per 100 miles for Sergei's old and new vehicle?

    Sergei's OLD vehicle: 100 miles/28 miles per gallon which is approximately 3.6 gallons per 100 miles

    Sergei's NEW vehicle: 100 miles/35 miles per gallon which is approximately 2.9 gallons per 100 miles

13. How much gasoline would he save driving 100 miles? 1,000 miles?

    3.6 gallons - 2.9 gallons = 0.7 gallons per 100 miles. This would be 7 gallons per 1000 miles.

14. What is the percent savings in gasoline? How does this compare to the answer found in question 10?

    Sergei saved 7 gallons from the original 36 gallons which is 7/36 or approximately 19%. These two amounts are approximately the same.

15. How much money did Sergei save?

    7 gallons times $2.50 per gallon is $17.50 savings.

Sergei's gasoline savings (7mpg) was higher than Laura's (5mpg). Yet, the amount of gasoline he saved was less than Laura's.

16. Explain why this is true.

    Sergei saved 7 gallons and Laura saved 10 gallons, even though his miles per gallon savings (7mpg) were higher than Laura's mpg (5mpg).

    There is a law of diminishing returns in that the higher the miles per gallon are, the less impact on the increase in gallons used. An increase of 5 mpg has a greater impact if you are getting 20 mpg as opposed to 30 miles per gallons to start with. The higher the mpg to start with, the less of an impact an increase in mpg will have in fuel economy savings.

Solutions to Practice Problems for Fuel Economy

1. Which driver do you think would be more upset? Why?
   Theresa is probably more upset since she is getting 4 mpg less than Shana who is getting only 2 mpg less.
2. What was the absolute difference in mpg for each vehicle?
   Theresa is getting 4 mpg less than indicated (40 mpg - 36 mpg = 4 mpg)
   Shana is getting 2 mpg less than indicated (25 mpg - 23 mpg = 2 mpg)
3. What was the percentage difference for each vehicle?
   Theresa: 4 mpg out of 40 mpg or 4/40 = 0.1 or 10%.
   Shana: 2 mpg out of 25 mpg or 2/25 = 0.08 or 8%.
4. How many more gallons of gas will Theresa buy because the rating was not accurate?
   Theresa expected: 1000 miles/40 mpg = 25 gallons
   Theresa got: 1000 miles/ 36 mpg is approximately 27.8 gallons
   Theresa will have to purchase 2.8 gallons more than she expected because the rating was not accurate.
5. How many more gallons of gas will Shana buy because the rating was not accurate?
   Shana expected: 1000 miles/25 mpg = 40 gallons
   Shana got: 1000 miles/23 mpg is approximately 43.5 gallons
   Shana will have to purchase 3.5 gallons more than she expected because the rating was not accurate.
6. Who is more affected by the incorrect rating?
   Shana is more affected by the incorrect rating because she will have to purchase 7 tenths of a gallon more every 1000 miles than Theresa will.