

# Germaine's Homework 

## Time Commitment

## Mathematical Goals

The student will use rates to determine the amount of time it takes to read a novel, informational text, and solve math problems.

The student will:

- Read a scenario and use rates to solve multi-step problems
- Understand there are two unit rates associated with each ratio
- Calculate rates
- Compare rates
- Work with rates in a meaningful context familiar to students


## Before the lesson, practice mental mathematics (5-10 minutes).

Number Talk Possibilities: Select two or three depending on student abilities.
State reading 20 pages in 5 minutes as three different ratios
State a reading rate of 20 pages in 5 minutes two different ways
State a unit rate of reading 20 pages in 5 minutes two different ways

- 20 pages in 5 minutes is the same as $\qquad$ pages in 1 minute
- 20 pages in 5 minutes is the same as 1 page in $\qquad$ minutes


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Germaine Studious has a lot of school work to complete over the next few days. He has 155 pages in a novel left to read for English class. He must solve 20 word problems for mathematics. Germaine also needs to read two chapters in an American history textbook to prepare for a quiz next week. There are 18 pages in each chapter. Currently, he is planning to spend seven hours over the weekend to complete the work but he is not sure this will be enough time.

"Charlene," he asked his older sister, "there's no way I can finish all of this work. I don't know how you manage!"
"Manage," she said, "is the operative word." "You work hard, but you don't work smart. What you need to do is budget your time. With a little planning, you'll be surprised at how much you can accomplish. Let's start by timing how long it takes you to read the next five pages of your novel."

Germaine took her advice. It took him 10 minutes to read five pages. Often, when talking about rate, the denominator is time. He determined his reading rate, R ,

$$
\mathrm{R}=5 \text { pages } / 10 \text { minutes }=0.5 \text { pages per minute } .
$$

"OK, he said. If I divide the number of pages by the number of minutes it took me to read them, I get a rate of .5 pages a minute. Great! Now how am I supposed to figure how long it will take me to do another 150 pages. I hate decimals!"
"Okay," his sister told him. "How about if you try to figure out how long it took you to read each page. I promise there won't be any decimals!"

10 minutes $/ 5$ pages $=2$ minutes per page.
With this number, it was easy to determine the time it would take him to finish the novel.

Time $=2$ minutes per page $\times 150$ pages $=300$ minutes.

Germaine was surprised that he would need 5 hours just to finish the novel. He felt if he focused he could read faster. He tried again and read the next five pages in only 8 minutes.

1. What is Germaine's reading rate when he focuses?
2. How long will it take him to read the remaining 145 pages with the faster reading rate?

Germaine then proceeded to solve the first pair of word problems in his mathematics text. It took him 14 minutes to solve the two problems.
3. How long will it take Germaine to solve the remaining 18 problems?
"Great!" Germaine said. At this rate, I should be able to finish the math pretty quickly. It looks like the problems get easier with practice. After all, second problem took less time to complete than the first."
"How long will it take Germaine to solve the remaining 18
 problems if, he used the second problem to estimate his rate of solving word problems?"
"Not so fast," Charlene said. "That hasn't been my experience with math. Take a few minutes to look over the whole set of problems. You're likely to find out that the problems at the end are more challenging - not less. You'll want to allow for more time to do those."
"You're right. Those last five problems have like a million steps. I'll need to figure twice as much time for that part of the assignment," Germaine told her.
4. How long will it take Germaine to solve these five problems?

5. How much time should Germaine plan for completing the math homework?

Germaine still needed to set aside time to read the history book and prepare for the quiz. The two chapters contained a total of 36 pages.
"The history should be a breeze," he said. "I already know that my reading rate is two minutes per page."
"Not so fast," his sister told him. "You may want to do another test run and recalculate."
"Spoilsport!" Germaine said, but he took her advice.
6. Why do you think the reading rate he found earlier should not be used in planning his American History studying?

Germaine read just two pages. It took him six minutes to read these two pages.
7. How long will it take Germaine to read the remaining 34 pages?
"That was really good," Charlene said to Germaine. "But I think you're forgetting something."
"Come on," Germaine said. "I read the pages and I timed myself. What's to forget?"
"Well," Charlene said, "You're not reading this for pleasure."
"I'll say."
"You're going to be tested on this material. Don't you think you should be taking...?"
"A break?" Germaine said hopefully. "Never mind, I was just kidding. You're right, Charlene. I need to allow for time to take notes while I read."

Germaine went back and took notes on the first two pages. It took him an additional five minutes.

8. How much time does Germaine need to add to his total for taking notes? Now what is the total time required to study for American history?

Germaine and Charlene's dad, Frank Lee Studious, works for Amazon.
"What were the two of you saying about note taking?" he asked.
"Dad, it's impossible. "At this rate I'll be in middle school the rest of my life," Germaine said.
"Let me help you," Mr. Studious said. "You're a smart boy and a hard worker. But working hard is often not as effective as working smart."
"That's what I said," Charlene exclaimed. Germaine said nothing.
"Here," their father said. "Let me show you how you can get by with a little help from your electronic friends. There is new technology available on electronic books that will enable you to take notes much more efficiently. Let me show you."

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Germaine tried it out. "Hey! You're right," he told his father. "I took me only two minutes to take notes on two whole pages. It looks like I may not have to spend the rest of my life in Middle School after all!"
"Good thing," said Mr. Studious with a chuckle, "because your mother and I don't intend to support you forever."
9. With this note taking technology, how much time could Germaine save? Now, what is the total time required to study for American history?

Project Idea: With a partner, find your reading rates for 5 pages of a novel, a history or science textbook, and calculate your rates for solving word problems. Use these rates to determine how much time it will take you to finish a novel, read a chapter in a textbook or solve 20 word problems.


## Practice Problems

According to https://wmich.edu/asc/files/IncreasingYourReadingRate.pdf reading is something that takes a lot of our time - between reading for classes, reading for fun and all the extra reading assignments for research projects and papers, there may be limited time for other extra-curricular activities. Increasing the speed of reading will help cut back on some of this time!


Different types of material read at different rates. The average reader reads a:
Newspaper at 400-600 wpm

Literature/Stories at 300-400 wpm

Textbooks at 240-300 wpm

To establish a base reading rate, use the following process. This can be used for comparison to new techniques only.

- Pick out a book.
- Read for 3 minutes marking both the beginning and an ending place.
- Multiply \# lines read by Avg. \# words per line then divide by 3 to get the reading rate.

1. What does wpm represent in the statements above?
2. Due to space restrictions, a newspaper reporter is limited to 5000 words for a local news story. How long will it take the average reader to read that article?
3. Mrs. Herstory, the social studies teacher, has assigned the class the reading of Chapter 6 in their textbook. The chapter consists of 18 pages with approximately 450 words on each page.
a. How long will it take the average reader to read Chapter 6?
b. Mrs. Herstory usually assigns not more than 30 minutes of reading homework per night. Does this assignment stay within her limit? Explain your thinking.
4. The English Department at Happy Valley School has adopted a new literature series. The 7th graders have been assigned "The Devil's Arithmetic." This novel contains 38,286 words. How long will it take the average reader to read the novel suggested? Explain your answer.
5. English classes at Happy Valley meet 5 days a week for 50 minutes per day. Of these 5 days, English teachers allow 20 minutes of reading time on Mondays, Wednesdays, and Fridays. Based on your answer to number 4 above, and if students only read the novel during English class, how many weeks should teachers plan for the unit on "The Devil's Arithmetic." Explain your thinking.

6. Use the process described above to establish your base reading rate for a new novel. Give your reading rate and explain how you calculated the average number of words per line.
7. In the process described above in the third bullet, why did you divide by 3 to get the reading rate?
8. Write a formula for calculating a reading rate if $m$ represents the number of minutes read, Let $l$ represents the number of lines read, and $w$ represents the average number of words per line.
9. Debbie noticed that she had read 150 lines in 3 minutes. What would Debbie's unit rate (lines per minute) be? How many seconds per line would this be?


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## Thinking Through a Lesson Protocol

## UNIT: Rates

Suggested time: Two to three class periods
Materials needed: A novel or history or science book to read to calculate reading rate

## Standards

6.RP.A.2: Understand the concept of $a$ unit rate $a / b$ associated with a ratio $a: b$ with $b$ not equal to 0 and use rate language in the context of a ratio relationship.
6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.A.3B: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

## Mathematical Practices:

MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP6: Attend to precision.
MP7: Look for and make use of structure.

| Setting up the Problem - Launch |  |
| :--- | :--- |
| Selecting tasks <br> and goal setting | (15 minutes) Ask students if they know their reading rate. Discuss <br> why the rate may be different for different types of reading. Provide <br> students with a common reading, one resource is suggested below. <br> Time all students reading for 2 minutes and have them mark where <br> they were when time was called. Have students count the number of <br> lines they read, calculate the average number of words per line as a <br> class, and use this to calculate their reading rate. Begin by <br> multiplying the number of lines they read, by the average number of <br> words per line (as determined by class), and then divide by 2. |
| Unlocking Children's Math Potential: 5 Research Results to <br> Transform Math Learning by Jo Boaler, Professor of Mathematics <br> Education, Stanford University https://bhi61nm2cr3mkdgk1dtaov18- <br> wpengine.netdna-ssl.com/wp-content/uploads/2017/03/teacher- <br> article-youcubed2.pdf |  |


| Monitoring student work - Explore |  |  |
| :--- | :--- | :--- |
| Part I - All Students | Qnd sequencing | Questions and Statements - <br> Monitoring |
| Strategies and misconceptions <br> - Anticipating | Who - Selecting <br> and |  |
| (10 minutes) Have students read <br> text and answer questions \#1-2 <br> individually. Share results in <br> whole group discussion. | Are there other rates that we could <br> find other than words per minute? <br> Is this a unit rate? How do you <br> know? Is there another way that <br> you could write a different unit <br> rate? |  |
| (15-20 minutes) Continue to <br> read text and answer questions <br> \#3-6 with a partner. |  | Is there a unit rate? What might it <br> be? What makes it a unit rate? |
| Have the whole group share out <br> their answers for 3-6. |  | Is there a unit rate? What might it <br> be? What makes it a unit rate? |
| (20-30 minutes) Continue <br> reading and with a partner <br> answer 7-11. <br> Share whole group. |  |  |


| Monitoring individual student work - Explore |  |  |
| :--- | :--- | :--- |
| Part II - Specific Groups of Students | Who - Selecting <br> and sequencing | Questions and Statements - <br> Monitoring |
| Strategies and misconceptions <br> - Anticipating | Mor |  |
| For off-task students or for <br> students that seem to be self- <br> conscious about you listening to <br> them share. | I am just listening or looking to find <br> out how you are working on the <br> problem. <br> This helps me think about what we <br> will do later. |  |
| For students that appear to be <br> stuck. <br> Also for when you are having a <br> difficult time understanding <br> their strategies. | Can you tell me a little about your <br> reading? <br> How would you describe the <br> problem in your own words? <br> What facts do you have? <br> Could you try it with simpler |  |
| numbers? |  |  |


| Managing the discussion - Summarize |  |
| :---: | :---: |
| Parts of discussion Connecting | Questions and statements - Connecting |
| Launching the discussion: <br> Select the problems in questions \#9-11 that students are struggling with or you wish to share out. | Will team1 start us off by sharing one way of working on this problem? <br> Please raise your hand when you are ready to share your solution. What did you do first when you were working on this problem? <br> Let's start by clearing up a few things about the problem. <br> Let's list some key parts in this problem. <br> What was unclear in the problem? |
| Eliciting and uncovering student strategies | Joe would you be willing to start us off? <br> What have you found so far? <br> Can you repeat that? <br> Can you explain how you got that answer? How do you know? <br> Walk us through your steps. Where did you begin? Can you show us? |
| Focusing on Mathematical Ideas | Can you explain why this is true? Does this method always work? <br> How is Bob's method similar to Kelly's method? <br> What do all the solutions have in common? <br> What would happen if I changed the numbers to $\qquad$ ? |
| Encouraging Interactions | Do you agree or disagree with Kahlil's idea? <br> What do others think? <br> Would someone be willing to repeat what Tom just said? <br> Would anyone be willing to add on to what Sue just said? |
| Concluding the Discussion | Can anyone tell me some of the big idea that we learned today? How would you explain what we learned today to a $5^{\text {th }}$ grader? <br> Some of the key points from our discussion today are . . . <br> Tomorrow we will continue our exploration of $\qquad$ beginning with the idea from today that $\qquad$ - |
| Post Lesson Notes | You may wish to assign the practice problems that you feel would benefit the students. |

## Solutions to Text Questions

1. What is Germaine's reading rate when he focuses?

When Germaine focuses his reading rate is 8 minutes for 5 pages or 1.6 minutes per page or $.675(5 / 8)$ pages per minute.
2. How long will it take him to read the remaining 145 pages with the faster reading rate?

It will take him 232 minutes or 3 hours and 52 minutes.
3. How long will it take Germaine to solve the remaining 18 problems?

Using 7 minutes per problem, it will take Germaine 126 minutes or 2 hours and 6 minutes to solve the remaining 18 problems.
4. How long will it take Germaine to solve the remaining 18 problems if he used the second problem to estimate his rate of solving word problems?

Using 6 minutes per problem, it will take Germaine 108 minutes of 1 hour 48 minutes to solve the remaining 18 problems.
5. How long will it take Germaine to solve these five problems?

If Germaine uses his slower rate ( 7 min per problem), it will take him 14 minutes to solve one problem or 70 minutes ( 1 hour 10 minutes) to solve the five problems. If he uses his faster rate ( 6 min per problem), it will take him 12 minutes per problem or 60 minutes ( 1 hour) to solve the five problems.
6. How much time should Germaine plan for completing the math homework?

If Germaine uses his slower rate ( 7 min per problem), it will take him (20 x $7=140$ ) for the easier problems and $(5 \times 14=70)$ for the more difficult problems for a total of $(140+70=210)$ or 3 hours 30 minutes for the 25 problem set.

If Germaine uses his faster rate ( 6 min per problem), it will take him (20 x $6=$ 120) for the easier problems and $(5 \times 12=60)$ for the more difficult problems for a total of $(120+60=180)$ or 3 hours for the 25 problem set.
7. Why do you think the reading rate he found earlier should not be used in planning his American History studying?

Germaine's early reading rate was determined from reading a novel. Reading for information from an American History book would require more comprehension. A different rate should be used.

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8. How long will it take Germaine to read the remaining 34 pages?

If Germaine reads 2 pages of American History in 6 minutes, his reading rate is 3 minutes per page or 1 page for every 3 minutes. It will take Germaine 34 x 3 or 102 minutes, 1 hour 42 minutes, to complete the American History remaining 34 pages.
9. How much time does Germaine need to add to his total for taking notes? Now what is the total time required to study for American history?

Germaine's note-taking at a rate of 2 pages in 5 minutes will result in a unit rate of 1 page per 2.5 minutes. For 36 pages the notetaking adds an additional 90 minutes or 1 hour 30 minutes to the American History studying. The total time for American History will then be

$$
\begin{gathered}
(\text { reading }+ \text { note-taking })=\text { total time or } \\
(1 \text { hour } 48 \text { minutes }+1 \text { hour } 30 \text { minutes })=3 \text { hours } 18 \text { minutes }
\end{gathered}
$$

10. With this note taking technology, how much time could he save? Now what is the total time required to study for American history?

The note-taking technology allows Germaine to take notes at a rate of 2 pages for every 2 minutes. The unit rate would be 1 page per minute so for 36 pages, notetaking should take him 36 minutes. Originally it was going to take him 90 minutes so he is saving 54 minutes!

Total time would then be:
(reading + note-taking $=$ total time or
(1 hour 48 minutes +36 minutes $=2$ hour 24 minutes
11. Were the seven hours Germaine planned to study enough for the work he needs to complete?

Germaine needs time to study for the three different content areas: the novel, mathematics, and American History. The rates and timing of each are as follows:

Novel: remaining 145 pages at a rate of $8 \mathrm{~min} / 5$ pages $=3$ hours 52 minutes
Math: $7 \mathrm{~min} / 1$ problem for 13 problems $+14 \mathrm{~min} / 1$ problem for 5 problems
$=2$ hours 41 minutes
American History: $6 \mathrm{~min} / 2$ pages for 34 pages $+36 \mathrm{~min} / 36$ pages

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$=2$ hours 24 minutes.
The TOTAL study time is 8 hours 57 minutes. Germaine's original estimate was that he would need 7 hours to study. In actuality, it is closer to 9 hours.

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## Solutions to Practice Problems

1. What does wpm represent in the statements above?
"wpm" means words per minutes
2. Due to space restrictions, a newspaper reporter is limited to 5000 words for a local news story. How long will it take the average reader to read that article?

If the average reader is reading at 400 wpm it will take 5000 words divided by 400 wpm or 12.5 minutes ( 12 minutes and 30 seconds) to read the article.

If the average reader is reading at 600 wpm it will take 5000 words divided by 600 wpm or 8.3333333 minutes ( 8 minutes and 20 seconds) to read the article.
3. Mrs. Herstory, the social studies teacher, has assigned the class the reading of Chapter 6 in their textbook. The chapter consists of 18 pages with approximately 450 words on each page.
a. How long will it take the average reader to read Chapter 6?

First to find the number of words multiply 18 pages by 450 words per page to get 8100 words. At 240 wpm, it will take 8100 words 240 wpm or 33.75 minutes ( 33 minutes and 45 seconds). At 300 wpm it will take 8100 words/ 300 wpm or 27 minutes. The average reader will take between 27 minutes and 33.75 minutes.
b. Mrs. Herstory usually assigns not more than 30 minutes of reading homework per night. Does this assignment stay within her limit?

Yes if the student reads at 300 wpm ; No if the student reads at 240 wpm. Explain your thinking. This assignment falls within the range of average readers, so some will be able to read in 30 minutes or less, others will take more than 30 minutes.
4. The English Department at Happy Valley School has adopted a new literature series. The 7th graders have been assigned "The Devil's Arithmetic." This novel contains 38,286 words. How long will it take the average reader to read the novel suggested? Explain your answer.

If the average reader rate is 300 wpm and the novel is 38,286 words, it will take 38,286 words divided by 300 wpm or approximately 128 minutes ( 2 hours and 8 minutes).

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If the average reader rate is 400 wpm and the novel is 38,286 words, it will take 38,286 words divided by 400 wpm or approximately 98 minutes ( 1 hour 38 minutes).
5. English classes at Happy Valley meet 5 days a week for 50 minutes per day. Of these 5 days, English teachers allow 20 minutes of reading time on Mondays, Wednesdays, and Fridays. Based on your answer to number 4 above, and if students only read the novel during English class, how many weeks should teachers plan for the unit on "The Devil's Arithmetic." Explain your thinking.

20 minutes of reading time on Mondays, Wednesdays, and Fridays is the equivalent of 60 minutes per week or 1 hour per week. In general terms the teachers should allow at least 2 weeks to accommodate most students. Average readers will need between 1.5 and 2 hours to read the novel.
6. Use the process described above to establish your base reading rate for a new novel. Give your reading rate and explain how you calculated the average number of words per line.

Reading rates will vary for students. How students calculate the average number of words per line will also vary. Some students may just pick one line and use it as an average, others may use 2 or 3 lines to calculate an average. Lines may be consecutive or randomly selected.
7. In the process described above in the third bullet, why did you divide by 3 to get the reading rate?

We divide by 3 to get the rate per minute. We read for 3 minutes.
8. Write a formula for calculating a reading rate if $m$ represents the number of minutes read, $l$ represents the number of lines read, and $w$ represents the average number of words per line.

$$
\text { Reading rate }=l w / m
$$

9. Debbie noticed that she had read 150 lines in 3 minutes. What would Debbie's unit rate (lines per minute) be?

Debbie's reading rate is (150 lines $/ 3$ minutes) 50 lines per minute.
How many seconds per line would this be?
50 lines in 60 seconds is the same as 1 line in 60/50 seconds or 1.2 seconds per line. This is the same as 1 line in 0.02 minutes.

