

Activity 2:
Making the grade
When 100% is possible

Mathematical Goals

The student will use percentages to determine what it will take to earn course grades for various courses.

In part I the student will:

- Read a scenario and use data presented in table format
- Calculate different percentages
- Calculate averages of two numbers
- Evaluate an expression
- Work with percentages in a meaningful context familiar to students

In part II, appropriate for students who have been introduced to basic algebra, the student will:

- Transition into the use of Algebra to answer a question
- Solve an algebraic equation

Before the lesson (5-10 minutes)

Put the paper and pencil down and practice some mental mathematics.

Number talk possibilities:

Select two or three depending on student abilities.

- Change $\frac{3}{4}$ to a decimal, then to a percent.
- Change $\frac{3}{10}$ to a decimal, then to a percent.
- Change $\frac{2}{5}$ to a decimal, then to a percent.
- Change $\frac{1}{8}$ to a decimal, then to a percent.
- Change $\frac{1}{3}$ to a decimal, then to a percent.
- Change $\frac{2}{3}$ to a decimal, then to a percent.



Making the grade – when 100% is possible

Danielle got eight out of ten questions right on a science test. The fraction of questions she got right was $\frac{8}{10}$. To change this fraction to an equal decimal, we can divide the numerator by the denominator: $8 \div 10 = 0.8$. Now, to find an equal percent, we multiply the decimal by 100%:

$$0.8 \times 100\% = 80\%.$$

Why change a decimal to a percent? Decimals have no natural range of values. They go from negative infinity to positive infinity. Danielle might live 100 meters from school, or 2,000 meters, or anything in between. The number 1.00 has no special meaning. However, 100% does have a special meaning. Danielle told her father she finished 15 homework problems. Her father does not know if she just started, was in the middle, or finished the whole assignment. But, if she told him she did 100% of her homework, he would know everything was done. If she told her father she did anything less than 100% of her homework, he would know she still had some work to do.

Frank's busy week at school

This was a busy test week for Frank. He had tests in math, science, and history. (See Table 1) Each test had a different number of questions. The math test had the fewest questions, because each one took a lot of time. He answered 12 out of the 15 questions correctly. The fraction he got

correct is $\frac{12}{15}$. Using his calculator, he found $\frac{12}{15} = 0.8$. Next, to multiply by 100%, he moved the decimal point two places to the right. His grade on the test was 80%.

	Math	Science	History
Correct Answers	12	20	19
Total Questions	15	40	20
Fraction Correct	$\frac{12}{15}$	$\frac{20}{40}$	$\frac{19}{20}$
Decimal	0.80	0.50	0.95
Percent	80%	50%	95%

Table 1 – Frank’s Exam Grades

The science test was all multiple choice questions. Frank answered many more questions right on the science test. He answered 20 questions correctly. However, there were 40 questions on the test. He only got half of the questions right. Thus, he earned 50% on this test. History is Frank’s favorite subject. The test was a mixture of multiple choice questions and short answers. He got 19 out of the 20 questions right.

Percentages make it easy to compare tests with a different number of questions.

1. Which subject does Frank need to work hardest on to improve his grade?
2. In which subject is there not much room for improvement?

One hundred percent is a perfect score. However, it is not the only standard that is used. For example, in Frank’s school, 65% is the passing grade on every test. Any student who fails a test must have a parent sign the test and return it to the teacher.



3. Will any of Frank’s tests need a parent’s signature? If so, which?

Randy's worries

Randy is not as studious as many of his classmates. His grades on the same tests are recorded in Table 2.

	Math	Science	History
Correct Answers	6	30	13
Total Questions	15	40	20

Table 2 – Randy's Exam Grades

4. Will Randy need to have any of his tests signed by a parent? If so, which?

Randy thinks he might fail math. He did very poorly on the test. This test is one of two for math. The two tests will be averaged to determine his grade in math. He wonders how well he will have to do on the second test to get his average above 65%.

5. What fraction of the questions on the math test did Randy get right?
6. What decimal is equal to the fraction of the questions Randy got it right? What percent is that?
7. If he scores 75% on the second test, will his average be more than 65%?
8. If he scores 98% on the second test, will his average be more than 65%?



Randy asked his friend Alberta to help him find the lowest score he'd need to get his average up to 65% or higher. Alberta began by showing Randy how to determine the average for different scores on the second exam. She told him to use x to represent the score of the second test.

Step 1: Add the new score to the first score, which was 0.4, to find the total of the two scores

$$0.4 + x$$

Step 2: Calculate the average by dividing by 2.

$$(0.4 + x) \div 2$$

Randy used step 1 and step 2 to set up a table and evaluate different scores on the second exam.

9. Help Randy by filling in the missing entries in Table 3 below.

Second test score x	Two scores added together $0.40 + x$	Average of two scores $(0.40 + x) \div 2$
0.50	$0.40 + .50 = .90$	$.90 \div 2 = .45$
0.55	$0.40 + 0.55 = 0.95$	$0.95 \div 2 = .475$
0.60		
0.65		
0.70		
0.75		

Table 3: Algebraic expressions calculate average

Part II – Algebra – Better than trial and error



By plugging in numbers, it is possible to see a pattern about the final grade. Randy soon got tired of the table. He still didn't know the exact answer. He asked Alberta if there was a shorter way to find the score he needed. In the above discussion, the representation $(0.40 + x) \div 2$ is called an algebraic expression. Alberta showed Randy how to use the algebraic expression to set up an equation to find the answer. Alberta said solving an equation would be shorter than his trial and error method.

An algebraic expression is a mathematical expression that consists of variables, constant numbers and mathematical operations. (Addition and multiplication are examples of mathematical operations.) The value of an algebraic expression can change as the value of the variable changes.

Alberta used \times the expression for the average of the two tests, but she made one small change. Instead of using the \div sign, she wrote the division as a fraction. Then she made an equation by putting the expression for the average equal to the required average of 65%. After she changed 65% to 0.65, the equation looked like this:

$$\frac{0.40 + x}{2} = 0.65$$

To solve this equation, Alberta said, “First we multiply both sides of the equation by the same number, 2.”

Randy said, “I see where the 2 came from, but why can we just multiply the left and right sides of the equation by 2?”

Alberta replied, “Imagine you had two equal amounts of money, like 2 quarters and 5 dimes. If you doubled both of them, wouldn’t you still have two equal amounts of money?” Randy agreed that they’d still be equal, so he did the multiplication.

$$2\left(\frac{0.40+x}{2}\right) = 2(0.65)$$

This got rid of the fraction on the left side of the equation. The new equation was

$$0.40 + x = 1.30$$

To solve this new equation, they looked for the number, x , to add to 0.40 so that the left hand side of the equation will total 1.30. To figure that out, they subtracted 0.40 from both sides.

$$0.40 + x = 1.30$$

$$-0.40 = -0.40$$

$$x = 0.90$$

This left only x to the left of the equal sign and 0.90 on the right. This means that Randy will need 0.90, or 90% on the second test to get an average of 65%.

The second test is going to have 20 questions. Alberta showed Randy how to use another equation to find out how many questions he needs to get right. If n is the number of questions he gets right, then the fraction answered correctly is $\frac{n}{20}$. That fraction must equal 0.90.

$$\frac{n}{20} = 0.90$$

$$20\left(\frac{n}{20}\right) = 20(0.90)$$

$$n = 20 \times 0.90 = 18$$



Randy was worried about how much he would have to improve just to earn a passing grade. He was assigned a tutor to work with him every day for the next week. He did much better on the second test. He got 17 out of 20 questions right.

10. What grade did Randy earn for the class?

11. How could he use these test results to try to convince the teacher to give him a passing grade for the class?

What if the fraction is not easy to change into a percent?

In the examples above, all of the fractions could easily be changed to an equal decimal. Gerald got 10 out of the 15 questions on the math exam right, or two-thirds of the questions. The equal decimal is 0.6666..., and the 6 repeats forever. This is 66.66...% with repeating 6s. In solving problems, we don't usually go beyond one decimal place. Thus, $\frac{2}{3}$ would be rounded to 66.7%. Often there is no need to be that accurate. The value can be rounded to the nearest whole percent. Gerald's teacher recorded his grade as 67%.

Gerald was also assigned a tutor. He wanted to do more than just pass math. He hoped to get an 85% average in math.

12. How high would he have to score to earn an 85% average?

Gerald's tutor thinks a more reasonable goal is to improve his average to 80%. Recall the final mathematics exam will contain 20 questions.

13. How many questions will Gerald need to get right to raise his average to 80% or higher?

14. Why do you think Gerald's tutor did not believe that 85% was a reasonable goal?

Project Idea: There are many ways that grades are calculated. Find out from your teacher how your grade is calculated in your math class and what grading scale is used. Calculate your current grade in your math class. To monitor your progress throughout the year, plot your current grade every two weeks on a graph.

Practice problems

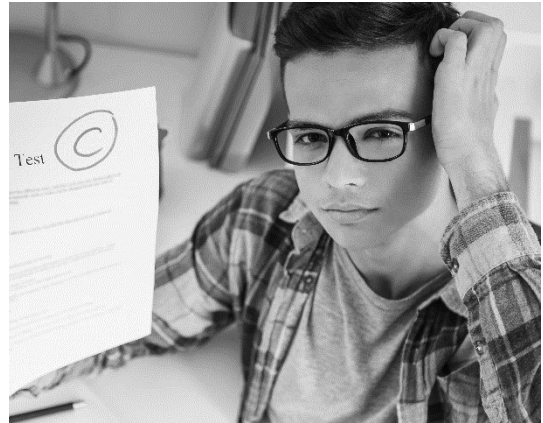
John has already received an 85% and a 75% on two of the three tests in his mathematics class. John's teacher uses the following grading scale.

Percentage	Grade
100 – 90	A
89 – 80	B
79 – 70	C
69 – 60	D

1. What is John's current average in the class?
2. If John gets 100% on the third exam, what grade will he receive in the class?
3. If John turns in a blank test and receives a zero on the third test, how will this affect his average?
4. Let x represent John's score on the third exam. Write a mathematical expression that can be used to calculate his average after receiving his score on the third exam.

Grade =?

5. John wants to be sure he earns at least a C grade in the class. Would he earn a C grade if he scored a 75 on the third exam? What if he scored a 65 on the third exam?



6. Write and solve an algebraic equation to determine the minimum percentage John needs on the third exam to receive a C in the class?
7. Determine the range of percentages John needs on the third test in order for him to receive a B in the class.
8. If the third test has a total of 20 questions, how many does John need to answer correctly to maintain his current average?

The local soccer organization plays 12 games in a season. Winning 60% or more of your games usually qualifies your team for the playoffs.



9. Your team, the Mustangs, has currently won 4 of the first 9 games. Is it possible for the Mustangs to make the playoffs? Why or why not?

10. Your friend's team, the Mavericks, have currently won 6 of the first 9 games. Is it possible for them to make the playoffs? Why or why not.