



Activity 2:

Teachers' guide

Thinking through a lesson protocol

Standards:

6.RP.A.3.C: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems.

7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematical Practices:

MP1: Make sense of problems and persevere in solving them.

MP2: Reason abstractly and quantitatively.

MP3: Construct viable arguments and critique the reasoning of others.

Setting up the problem - Launch	
Selecting tasks/goal setting	(5 minutes) Briefly discuss or ask students in a whole group setting for ideas when percentages are used in place of a decimal or a fraction. Then have students read individually the first two paragraphs of the scenario, stopping at Frank's Busy Week at School. With a partner talk about what you think is the big idea in the reading.
Questions	Can you think of a time that percentages are used? When are they used in place of a decimal or a fraction? Why do you think they are used instead of a decimal or a fraction?

Monitoring student work - Explore		
Strategies and misconceptions- Anticipating	Who - Selecting and sequencing	Questions and statements - Monitoring
(10 minutes) Have students look at Table 1: Frank's Exam Grades and note two pieces of information found in the table. Have students share ideas in whole group setting while teacher records on board.		What are two pieces of information that you found in Table 1? (Teacher records on board).
Then read Frank's Busy Week at School and with a partner answer questions #1-3. Share out the results of questions #1-3.		As students share out their findings for questions 1-3 make sure that they justify their reasoning.
(30-40 minutes) In small groups have students continue to read Randy's Worries and answer questions #4-9. Monitor student work and check for understanding as the group's progress.		
(30-40 minutes) Read Part II - Algebra as a whole class. Carefully explain the Addition and Multiplication Properties of Equality while solving the equations. Have students solve questions #10-14 with a partner. Check for understanding while students are working. Share results as time allows.		Depending on students' prior knowledge of solving algebraic equations the time spent on this section may vary. Spend more time if this is their first exposure, less if they are comfortable with solving simple equations. Multiplying both sides of an equation by a number is an example of the Multiplication Property of Equality.

Monitoring individual student work - Explore		
Strategies and misconceptions- Anticipating	Who - Selecting and sequencing	Questions and Statements - Monitoring
For off-task students or for students that seem to be self-conscious about you listening to them share.		<p>I am just listening or looking to find out how you are working on the problem.</p> <p>This helps me think about what we will do later.</p> <p>What do you think is the Big Idea in the Introduction to Percentages reading?</p>
For students that appear to be stuck. Also for when you are having a difficult time understanding their strategies.		<p>Can you tell me a little about your reading?</p> <p>How would you describe the problem in your own words?</p> <p>What facts do you have?</p> <p>Could you try it with simpler numbers?</p>
For students that want to ask you questions, these are ways to uncover their thinking and judge to what extent you want to respond.		<p>Tell me what you've thought about so far.</p> <p>What do you know?</p> <p>Why are you interested in more information about that?</p> <p>Let me say a little about that part.</p>

Managing the discussion – Summarize	
Parts of discussion - Connecting	Questions and statements - Connecting
Launching the discussion: Select the problems in questions #10-14 that students are struggling with or you wish to share out.	Will team 1 start us off by sharing one way of working on this problem? Please raise your hand when you are ready to share your solution. What did you do first when you were working on this problem? Let's start by clearing up a few things about the problem. Let's list some key parts in this problem? What was unclear in the problem?
Eliciting and uncovering student strategies	Joe would you be willing to start us off? What have you found so far? Can you repeat that? Can you explain how you got that answer? How do you know? Walk us through your steps. Where did you begin? Can you show us?
Focusing on mathematical ideas	Can you explain why this is true? Does this method always work? How is Bob's method similar to Kelly's method? What do all the solutions have in common? What would happen if I changed the numbers to ____?
Encouraging interactions	Do you agree or disagree with Kahlil's idea? What do others think? Would someone be willing to repeat what Tom just said? Would anyone be willing to add on to what Sue just said?
Concluding the discussion	Can anyone tell me some of the big ideas that we learned today? How would you explain what we learned today to a 5 th grader? Some of the key points from our discussion today are . . . Tomorrow we will continue our exploration of _____ beginning with the idea from today that _____.
Post lesson notes	You may wish to assign the practice problems that you feel would benefit the students.

Solutions: Making the grade – When 100% is possible

1. Which subject does Frank need to work hardest on to improve his grade?

Science: His first exam grade was the lowest, 50%

2. In which subject is there not much room for improvement?

History: His first exam grade was 95% which is almost perfect.

3. Will any of Frank's tests need a parent's signature? If so, which?

Science: His first exam grade was less than 65%

4. Will Randy need to have any of his tests signed by a parent? If so, which?

Math Grade = $6 / 15 = 0.4 \rightarrow 40\%$

Parent's signature required

Science Grade = $30 / 40 = 0.75 \rightarrow 75\%$

No signature required

History Grade = $13 / 20 = 0.65 \rightarrow 65\%$

No signature required

5. What fraction of the questions on the math test did Randy get right?

$6 / 15$

6. What decimal is equal to the fraction of the questions Randy got right? What percent is that?

$6 / 15 = 0.4$

$0.4 \times 100\% = 40\%$

7. If he scores 75% on the second test, will his average be more than 65%?

No $(40\% + 75\%) / 2 = 57.5\%$

8. If he scores 98% on the second test, will his average be more than 65%?

Yes $(40\% + 98\%) / 2 = 69\%$

9. Help Randy by filling in the missing entries in Table 3 below.

Second test score x	Two scores added together $0.40 + x$	Average of two scores $(0.40 + x) \div 2$
0.50	$0.40 + 0.50 = .90$	$0.90 \div 2 = .045$
0.55	$0.40 + 0.55 = 0.95$	$0.95 \div 2 = 0.475$
0.60	$0.40 + 0.60 = 1.00$	$1.00 \div 2 = 0.50$
0.65	$0.40 + 0.65 = 1.05$	$1.05 \div 2 = 0.525$
0.70	$0.40 + .70 = 1.10$	$1.10 \div 2 = 0.55$

Table 3: Algebraic expressions calculate average

10. What grade did Randy earn for the class?

$$17/20 = 0.85 \rightarrow 85\%$$

$$\text{Average } (40\% + 85\%) / 2 = 62.5\%$$

11. How could he use these test results to try to convince the teacher to give him a passing grade for the class?

- *One argument is that the two exams were not equally sized. If the two exams were combined the total number of questions is 35. He answered 6 questions correctly on the first exam and 17 on the second exam. In total he correctly answered 23 of 35 questions.*
 $23 / 35 = 65.7\%$ *which is a passing grade*
- *He could also argue the teacher should reward him for improving significantly from the first to the second exam.*

12. How high would he have to score to earn an 85% average?

$$(0.67 + x) / 2 = 0.85$$

$$(0.67 + x) = 0.85(2) = 1.7$$

$$0.67 + x - 0.67 = 1.7 - 0.67$$

$$x = 1.03 \rightarrow 103\%$$

13. Why do you think Gerald's tutor did not believe that 85% was a reasonable goal?

It is not possible to score more than 100% on an exam.

14. How many questions will Gerald need to get right to raise his average to 80% or higher?

First calculate the percentage score that is needed.

$$(0.67 + x)/2 = 0.8$$

$$(0.67 + x) = 0.8(2) = 1.6$$

$$0.67 + x - 0.67 = 1.6 - 0.67$$

$$x = 0.93 \rightarrow 93\%$$

He will need to score at least 93%. The exam has 20 questions.

$$\text{Number of questions correct} = 0.93 \times 20 = 18.6$$

He will need to answer 18.6 questions correctly but there are no fractional questions.

That means he will need to answer at least 19 questions correctly to achieve his goal of at least an 80% grade.

Solutions to practice problems

1. What is John's current average in the class?

$$(85\% + 75\%) / 2 = 80\%$$

Students might reason that splitting the difference between 85% and 75% would be 80%.

2. If John gets 100% on the third exam, what grade will he receive in the class?

$$(85\% + 75\% + 100\%) / 3 = 86.666...\%$$

According to the grading scale, 80-89% is a B.

John will receive a B in the class.

3. If John turns in a blank test and receives a zero on the third test, how will this affect his average?

$$(75\% + 85\% + 0\%) / 3$$

$$160\% / 3 = 53.333...\%$$

This will bring his current average of 80% down to a failing grade of 53.333...%.

4. Is it possible for John to get an A in the class? Why or why not?

It is not possible for John to get an A in the class. Receiving a perfect score, 100% on the final test only earns him an 87% (if the teacher rounds up).

5. What is the minimum percentage John needs on the third exam to receive a C in the class?

$$(85\% + 75\% + x\%) / 3 = 70\%$$

$$(160\% + x\%) / 3 = 70\%$$

$$160\% + x\% = 210\%$$

$$x\% = 50\%$$

6. Determine the range of percentages John needs on the third test in order for him to receive a B in the class.

To receive a B in the class, the minimum amount that John can receive on the third test is an 80%.

$$(85\% + 75\% + x\%) / 3 = 80\%$$

$$(160\% + x\%) / 3 = 80\%$$

$$160\% + x\% = 240\%$$

$$x\% = 80\%$$

$$(85\% + 75\% + x\%) / 3 = 89\%$$

$$(160\% + x\%) / 3 = 89\%$$

$$160\% + x\% = 267\%$$

$$x\% = 107\%$$

Since the maximum score is 100%, 107% is not possible. If you refer back to number 2, the maximum score of 100% was earned on the third test. The result was an average of 87%.

7. If the third test has a total of 20 questions, how many does John need to answer correctly to maintain his current average?

$$x / 20 = 80\%$$

$$x = 80\%(20)$$

$$x = 0.80(20)$$

$$x = 16$$

John will need to answer 16 out of the 20 questions correctly.

8. Your team, the Mustangs, has currently won 4 of the first 9 games. Is it possible for the Mustangs to make the playoffs? Why or why not?

The Mustangs currently have 4 wins / 9 games or have won 44.444...% of the games.

If the Mustangs continue to win, their record would improve to:

5 wins / 10 games; 6 wins / 11 games; and finally 7 wins / 12 games.

50% 55.545...% 58.333...%

The Mustangs would typically NOT QUALIFY for the playoffs as the highest percent they could have would be 58%.

9. Your friend's team, the Mavericks, have currently won 6 of the first 9 games. Is it possible for them to make the playoffs? Why or why not.

Scenario 1: If the Mavericks win their three remaining games, their record will be 9 wins / 12 games or a 75% record. They would QUALIFY for the playoffs.

Scenario 2: If the Mavericks win two of their three remaining games, their record will be 8 wins / 12 games or a 66.666...% record. They would QUALIFY for the playoffs.

Scenario 3: If the Mavericks win one of their three remaining games, their record will be 7 wins / 12 games or a 58.333...% record. They would NOT QUALIFY for the playoffs.

Scenario 4: If the Mavericks don't win any of their three remaining games, their record will be 6 wins / 12 games or a 50% record. They would NOT QUALIFY for the playoffs.